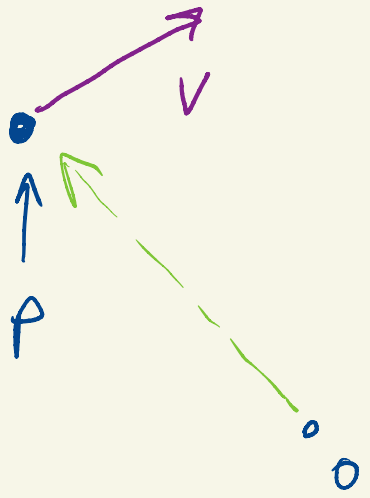
$x + \bar{c}y$

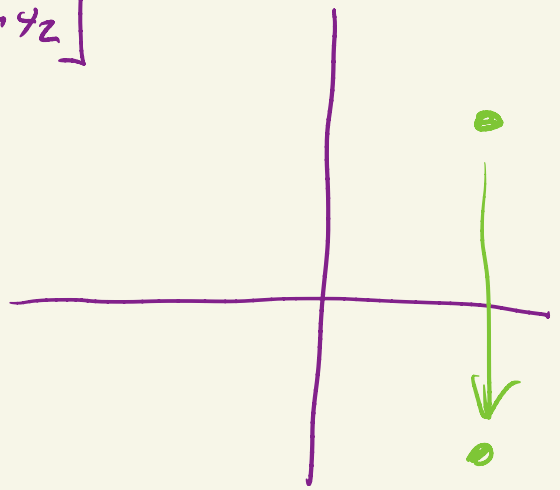
$$\begin{bmatrix} x & -y \\ y & x \end{bmatrix}$$

$w = p + sv$

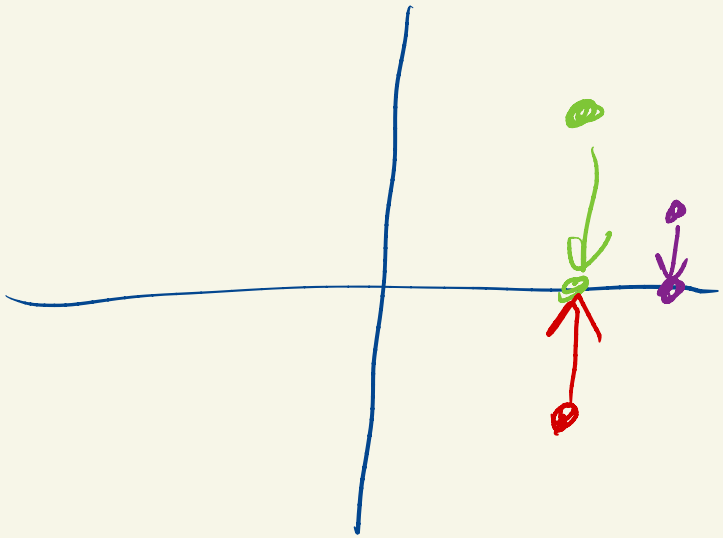


$$M = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad M \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ -x_2 \end{bmatrix}$$

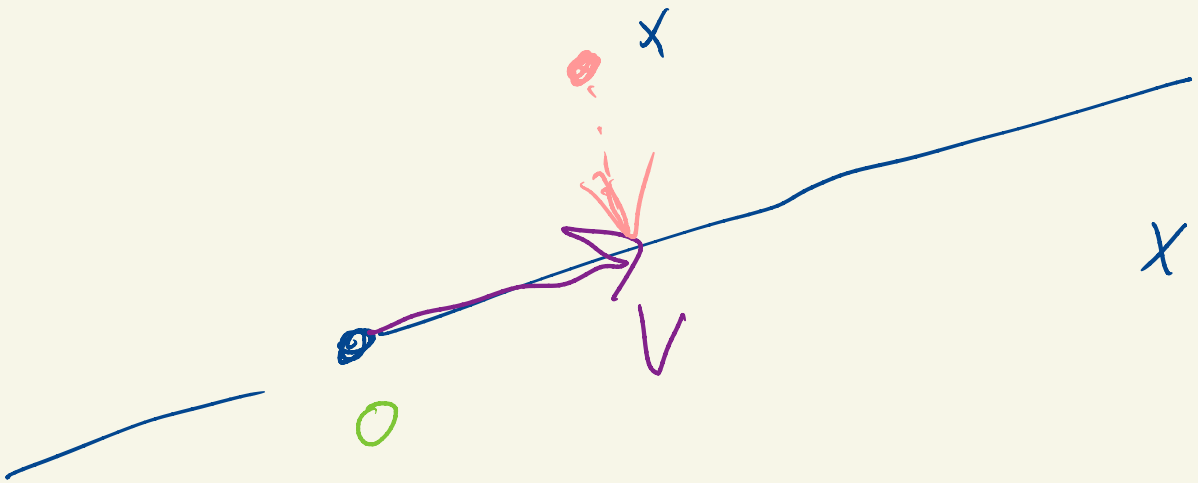
Reflection about x-axis



$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 \\ 0 \end{bmatrix}$$



"Orthogonal projection onto x-axis"



$$x \longmapsto \frac{x^T v}{\|v\|^2} v$$

$$M = \frac{1}{\|v\|^2} \begin{bmatrix} v_1 v_1 & v_1 v_2 \\ v_2 v_1 & v_2 v_2 \end{bmatrix}$$

$$M_x = \frac{1}{\|v\|^2} \begin{bmatrix} v_1 v_1 & v_1 v_2 \\ v_2 v_1 & v_2 v_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{\|v\|^2} \begin{bmatrix} v_1 \overbrace{(v_1 x_1 + v_2 x_2)}^{v^T x} \\ v_2 v_1 x_1 + v_2 v_2 x_2 \end{bmatrix}$$

$$= \frac{v^T x}{\|v\|^2} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \frac{v^T x}{\|v\|^2} v$$

Down sampling

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} \rightarrow \begin{bmatrix} x_1 \\ x_3 \\ x_5 \end{bmatrix}$$

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$$3 \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_3 \\ x_5 \end{bmatrix}$$

Smoothing

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \mapsto \begin{bmatrix} (x_1+x_2)/2 \\ (x_1+x_2+x_3)/3 \\ (x_2+x_3+x_4)/3 \\ (x_3+x_4+x_5)/3 \\ (x_4+x_5)/2 \end{bmatrix}$$

$$\begin{bmatrix} 1/2 & 1/2 & 0 & 0 & 0 \\ 1/3 & 1/3 & 1/3 & 0 & 0 \\ 0 & 1/3 & 1/3 & 1/3 & 0 \\ 0 & 0 & 1/3 & 1/3 & 1/3 \\ 0 & 0 & 0 & 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} (x_1+x_2)/2 \\ (x_1+x_2+x_3)/3 \\ \vdots \\ \vdots \\ \vdots \end{bmatrix}$$

Convolution $\frac{1}{3}\ \frac{1}{3}\ \frac{1}{3}$ ⟶ mostly just averaging

$a = (a_1, a_2, a_3)$

$$\begin{bmatrix} a_1 & 0 & 0 & 0 \\ a_2 & a_1 & 0 & 0 \\ a_3 & a_2 & a_1 & 0 \\ 0 & a_3 & a_2 & a_1 \\ 0 & 0 & a_3 & a_2 \\ 0 & 0 & 0 & a_3 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = \begin{bmatrix} a_1 b_1 \\ a_2 b_1 + a_1 b_2 \\ a_3 b_1 + a_2 b_2 + a_1 b_3 \\ a_3 b_2 + a_2 b_3 + a_1 b_4 \\ a_3 b_3 + a_2 b_4 \\ a_3 b_4 \end{bmatrix}$$

$(b_1, b_2, b_3, \ldots, b_n)$

(a_3, a_2, a_1)

(a_3, a_2, a_1)

window w/ weights

$$(a_3x^2 + a_2x + a_1)(b_3x^2 + b_2x + b_1)$$

$$= a_3b_3x^4 + (a_2b_3 + a_3b_2)x^3$$

$$+ (a_3b_1 + a_2b_2 + a_1b_3)x^2$$

$$+ (a_1b_2 + a_2b_1)x$$

$$+ a_1b_1$$

(b_1, b_2, b_3) ← b

(a_3, a_2, a_1)

$a * b$

↑ a

"convolution of a with b"

$$(a_1b_1,\ a_1b_2 + a_2b_1,\ a_3b_1 + a_2b_3 + a_1b_3,\ a_2b_3 + a_3b_2,\ a_3b_3)$$

$a \in \mathbb{R}^n$

$b \in \mathbb{R}^m$

$n + m - 1$

Linear Functions

$f(x) = c^T x$

$f: \mathbb{R}^n \to \mathbb{R}$

$$f(x+y) = f(x) + f(y)$$
$$f(cx) = c\, f(x)$$

] linearity

$f: \mathbb{R}^n \to \mathbb{R}^m$

$$f(\alpha x + \beta y) = \alpha f(x) + \beta f(y) \quad \Leftarrow \text{superposition}$$