Applications:

$$\overset{A}{\begin{bmatrix} a_1 & \cdots & a_n \end{bmatrix}} \overset{x}{\begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}} = \underbrace{x_1 a_1 + \cdots + x_n a_n}$$

audio

audio time signal

$$\begin{bmatrix} a_1 & a_2 \end{bmatrix} \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} = \frac{1}{2} a_1 + \frac{1}{2} a_2 = \frac{1}{2}(a_1 + a_2)$$

$$\begin{array}{cc} & n+1 \\ n & \begin{bmatrix} -1 & 1 & 0 & \cdots & & 0 \\ 0 & -1 & 1 & 0 & \cdots & 0 \\ 0 & 0 & -1 & 1 & 0 \cdots & 0 \\ \vdots & \vdots & & \ddots & & 0 \\ 0 & \cdots & & 0 & -1 & 1 \end{bmatrix} \end{array} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n+1} \end{bmatrix} = \begin{bmatrix} x_2 - x_1 \\ x_3 - x_2 \\ \vdots \\ x_{n+1} - x_n \end{bmatrix}$$

"discrete derivative"

temperatures at 1 minute intervals

AAPL

corporations

months

1

⋮

12

-16

A

x

# of shares in a portfolio

$ change in share price that month

net change in portfolio value that month

Ax

$t_1, t_2, \ldots, t_m$

$$\begin{array}{c} n+1 \\ \begin{bmatrix} 1 & t_1 & t_1^2 & \cdots & t_1^n \\ 1 & t_2 & t_2^2 & \cdots & t_2^n \\ \vdots & & & & \\ 1 & t_m & t_m^2 & \cdots & t_m^n \end{bmatrix} \end{array} \begin{bmatrix} c_0 \\ c_1 \\ \vdots \\ c_n \end{bmatrix} = \begin{bmatrix} c_0 + c_1 t_1 + c_2 t_1^2 + \cdots + c_n t_1^n \\ c_0 + c_1 t_2 + c_2 t_2^2 + \cdots + c_n t_2^n \\ \vdots \\ c_0 + c_1 t_m + \cdots + c_n t_m^n \end{bmatrix}$$

Vandermonde matrix

$$p(t) = c_0 + c_1 t + \cdots + c_n t^n$$

$$\begin{bmatrix} p(t_1) \\ p(t_2) \\ \vdots \\ p(t_m) \end{bmatrix}$$

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n-vector

$$\underset{m \times n}{\bigcirc}\; x = \underset{m\text{-vector}}{\bigcirc}$$

$$\underset{n \times n}{I}\;\underset{n\text{-vector}}{x} = x \qquad I = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & 0 \cdots & 0 \\ & & \ddots & 0 \\ 0 & & & \end{bmatrix}$$

$$I = \begin{bmatrix} e_1 & e_2 & \cdots & e_n \end{bmatrix}$$

I acts like the number 1 for multiplication

$$I x = \begin{bmatrix} e_1 & \cdots & e_n \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = x_1 e_1 + x_2 e_2 + \cdots + x_n e_n$$

$$= x$$

$$\underbrace{A e_k}_{k^{th}\text{ column of } A} = \begin{bmatrix} a_1 & a_2 & \cdots & a_n \end{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \leftarrow \text{slot } k$$

$$= 0 \cdot a_1 + 0 a_2 \cdots 1 a_k + \cdot 0 a_{k+1} \cdots + 0 a_n = a_k$$

$\vec{1}$

$$\underset{m \times n}{A} \ \underset{n \text{ vector}}{\vec{1}} = \begin{bmatrix} a_1 & \cdots & a_n \end{bmatrix} \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} = 1 a_1 + 1 a_2 + \cdots + 1 a_n$$

$$= a_1 + a_2 + \cdots + a_n$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 1 \begin{bmatrix} 1 \\ 4 \end{bmatrix} + 1 \begin{bmatrix} 2 \\ 5 \end{bmatrix} + 1 \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1+2+3 \\ 4+5+6 \end{bmatrix}$$

rules for linear functions

$$\left.\begin{aligned} A(x+y) &= Ax + Ay \\ A(cx) &= c(Ax) \end{aligned}\right] \rightarrow$$

"superposition"

$$A(\alpha x + \beta y) = A(\alpha x) + A(\beta y)$$

$$= \alpha(Ax) + \beta(Ay)$$

$$(A+B)x = Ax + Bx$$

$$(cA)x = c(Ax)$$

Given a $m \times n$ matrix A

we have an associated map $\mathbb{R}^n \to \mathbb{R}^m$

$$\underset{\substack{\uparrow \\ \mathbb{R}^n}}{x} \longmapsto \underset{\substack{\uparrow \\ \mathbb{R}^m}}{\underbrace{Ax}} \qquad \overbrace{\underset{\text{signals}}{A}\ \underset{\text{weights}}{x}}^{\text{mix}}$$

e.g.: share price changes | assets $\longrightarrow$ portfolio net change value by month

audio signals | weights $\longrightarrow$ mixed signal

I | vector $\longrightarrow$ same thing

Θ, angle

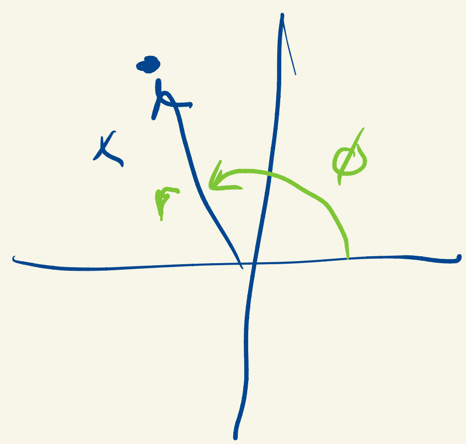
$$R_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$x = \begin{bmatrix} r\cos\phi \\ r\sin\phi \end{bmatrix}$$

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

$$\sin(A+B) = \cos(A)\sin(B) + \cos(B)\sin(A)$$



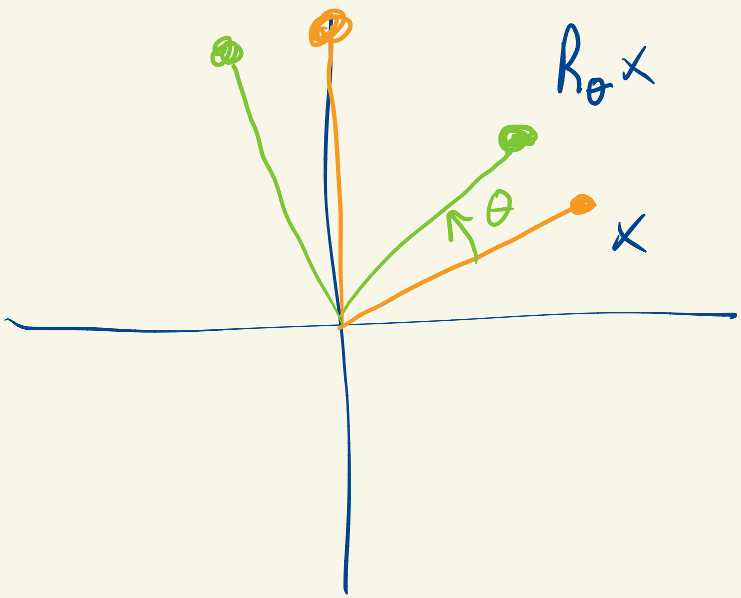
$$R_\theta x = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} r\cos\phi \\ r\sin\phi \end{bmatrix}$$

$$= r \begin{bmatrix} \cos\theta\cos\phi - \sin\theta\sin\phi \\ \sin\theta\cos\phi + \cos\theta\sin\phi \end{bmatrix}$$

$$= r \begin{bmatrix} \cos(\phi+\theta) \\ \sin(\phi+\theta) \end{bmatrix}$$

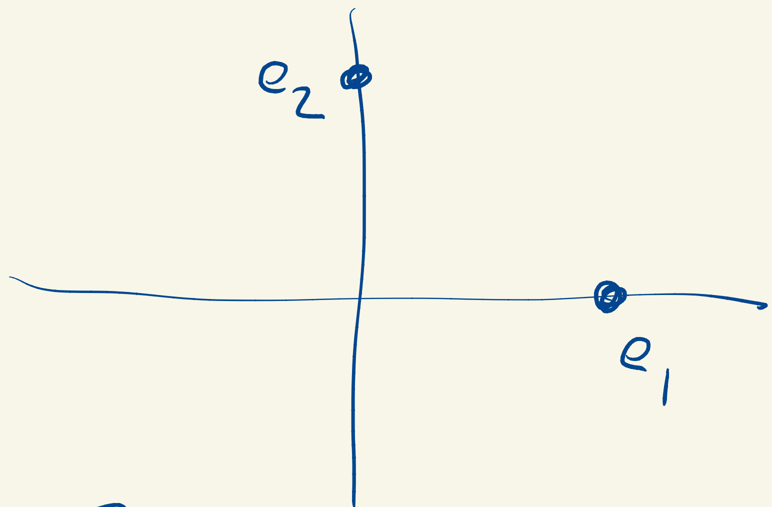
$$x = r \begin{bmatrix} \cos\phi \\ \sin\phi \end{bmatrix} \qquad R_\theta x = r \begin{bmatrix} \cos(\phi+\theta) \\ \sin(\phi+\theta) \end{bmatrix}$$

R_θ represents rotation of the plane by angle θ



$$x \longmapsto R_\theta x$$

rotate x by angle θ.



$$R_{\frac{\pi}{2}} = \begin{bmatrix} \cos(\pi/2) & -\sin(\pi/2) \\ \sin(\pi/2) & \cos(\pi/2) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

$$R_{\pi/2}\, e_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = e_2$$

$$R_{\pi/2}\, e_2 = \begin{bmatrix} -1 \\ 0 \end{bmatrix} = -e_1$$

$x + \bar{i}y$

$$\begin{bmatrix} x & -y \\ y & x \end{bmatrix}$$