Operations With Matrices

transpose

① Transpose $\quad T \qquad x^T y$

$$x = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \qquad x^T = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

$3 \times 1 \qquad 1 \times 3$

exchanging columns for rows

$$(A^T)_{ij} = A_{ji}$$

swapping cols and rows

$$\begin{bmatrix} 2 & 0 & 0 \\ 4 & 1 & 0 \\ 5 & 6 & 3 \end{bmatrix}^T = \begin{bmatrix} 2 & 4 & 5 \\ 0 & 1 & 6 \\ 0 & 0 & 3 \end{bmatrix} \qquad (A^T)_{23} = A_{32}$$

lower triangular (all 0's above diagonal) $\qquad$ upper triangular

$$\begin{bmatrix} a_1, \dots, a_k \end{bmatrix}^T = \begin{bmatrix} a_1^T \\ a_2^T \\ \vdots \\ a_k^T \end{bmatrix}$$

$a_1, \dots, a_k$ n-vectors

A is $m \times n$

A^T is $n \times m$

$$\begin{bmatrix} a_1^T \\ \vdots \\ a_k^T \end{bmatrix}^T = \begin{bmatrix} a_1 & a_2 & \cdots & a_k \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

$$(A^T)^T = A$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^T = \begin{bmatrix} A^T & C^T \\ B^T & D^T \end{bmatrix}$$

Adding matrices:

same shape $m \times n$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 7 \\ 2 & 0 & 6 \end{bmatrix} = \begin{bmatrix} 10 & 2 & 10 \\ 6 & 5 & 12 \end{bmatrix}$$

$$(A+B)_{ij} = A_{ij} + B_{ij}$$

Multiplication by scalars

$c \in \mathbb{R}$

$$2 \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \end{bmatrix}$$

$d, c \in \mathbb{R}$ $\quad A, B$ $m \times n$ matrices

$$c(A+B) = cA + cB$$

$$(c+d)A = cA + dA$$

$$c(dA) = (cd)A$$

$$2a - 3b + 5c = 3$$
$$-a + b + 2c = 5$$

$$a\begin{bmatrix} 2 \\ -1 \end{bmatrix} + b\begin{bmatrix} -3 \\ 1 \end{bmatrix} + c\begin{bmatrix} 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 & 5 \\ -1 & 1 & 2 \end{bmatrix}\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$A \qquad x \qquad b$

$$Ax = b$$

Matrix-vector multiplication

$$\begin{array}{ccc} & n & \\ m & \begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \end{bmatrix} & \begin{bmatrix} \quad \\ \\ \\ \\ \end{bmatrix} n \end{array}$$

must match

1) Column perspective

$$\begin{bmatrix} a_1 \cdots & a_n \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = x_1 a_1 + x_2 a_2 + \cdots + \overset{\text{number}}{x_n} \, \overset{\text{vector}}{a_n}$$

$$\begin{bmatrix} 2 & -3 & 5 \\ -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = 1 \begin{bmatrix} 2 \\ -1 \end{bmatrix} + 2 \begin{bmatrix} -3 \\ 1 \end{bmatrix} + 1 \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

2) Row perspective

$$\begin{array}{cc} & n \\ m & \begin{bmatrix} b_1^T \\ \vdots \\ b_m^T \end{bmatrix} \end{array} x = \begin{bmatrix} b_1^T x \\ b_2^T x \\ \vdots \\ b_m^T x \end{bmatrix}$$

<u>n vector</u>

$b_1, \ldots, b_m$ n-vectors

$$\begin{bmatrix} 2 & -3 & 5 \\ -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} (2,-3,5)^T (1,2,1) \\ (-1,1,2)^T (1,2,1) \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$\boxed{(\underset{m \times n}{A}\ \underset{n\text{-vector}}{x})_i = \sum_{j=1}^{n} A_{ij} x_j}$$

result: m-vector