Matrizes:

dimensions of the matrix

$\underset{\text{rows}}{3} \times \underset{\text{columns}}{4}$ matrix

$$A = \underbrace{\begin{bmatrix} 2 & 1 & 3 & 4 \\ 3 & -1 & 2 & 6 \\ 4 & 1 & 9 & 3 \end{bmatrix}}_{4 \text{ columns}} \Big] \ 3 \text{ rows}$$

$$A_{\underset{\text{row}}{2}\,\underset{\text{column}}{4}} = 6, \qquad A_{32} = 1 \qquad A_{\underset{\text{row}}{i}\,\underset{\text{column}}{j}}$$

vectors can be thought of as a species of matrix

$$\begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix} \leftarrow \text{vector of dim } 3$$

3×1 matrix

$\begin{bmatrix} 4 & 1 & 9 & 3 \end{bmatrix}$ isn't a vector

1×4 "row vector" $1 \times k$ matrix

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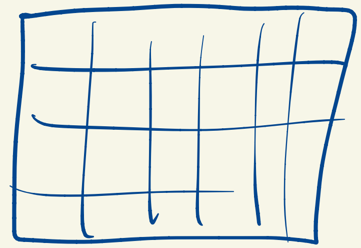
examples

1) Images

6 columns

4 rows



4×6 matrix with a number for each pixel

2) Lists of vectors

$a_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ $a_2 = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ $a_3 = \begin{bmatrix} 9 \\ 3 \end{bmatrix}$

$$A = \begin{bmatrix} 2 & 4 & 9 \\ 1 & 1 & 3 \end{bmatrix}$$

$$\begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix}$$

"block matrix notation"

$$A \longrightarrow Q$$

Gram Schmidt

$$\begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} \qquad \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix}$$

e.g.

$$\begin{array}{c|cccc} & 1 & 2 & 3 & \cdots\; 12 \\ \hline \text{Palmer} & & & \bullet & \\ \text{Jerrod} & & & & \\ \text{Arch} & & & & \\ \text{Haines} & & & & \end{array}$$

4×12 matrix

avg daily temp for month 3
in Palmer

example: contingency tables

dog ownership

cat ownership	0	1	2	3	4+
0					
1				$\square$	
2					
3					
4+					

→ # of people with 3 dogs, 1 cat

Block matrices

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 5 & 7 & 9 \\ 6 & 8 & 10 \end{bmatrix} \rightarrow 2 \times 3$$

$$C = \begin{bmatrix} 11 & 12 \end{bmatrix} \qquad D = \begin{bmatrix} 13 & 14 & 15 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \left[\begin{array}{cc:ccc} 1 & 2 & 5 & 7 & 9 \\ 3 & 4 & 6 & 8 & 10 \\ \hdashline 11 & 12 & 13 & 14 & 15 \end{array}\right]$$

$a_1, \dots, a_k \qquad \begin{bmatrix} a_1 & \cdots & a_k \end{bmatrix}$

row vectors $\underbrace{b_1, \dots, b_k}_{\substack{1 \times n \text{ matrices} \\ \text{(row vectors)}}}$ $\qquad \begin{bmatrix} b_1 \\ \vdots \\ b_k \end{bmatrix} \leftarrow k \times n$

A $\underline{n \times m}$ matrix is square if $m = n$

is tall if $n > m$

is wide if $n < m$

An important square matrix

diagonal

$$I = \underbrace{\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & \ddots & & \vdots \\ 0 & 0 & & \ddots & 0 \\ \vdots & \vdots & & & 1 \\ 0 & 0 & \cdots & 0 & \end{bmatrix}}_{n} \Bigg\} n$$

$$I_{ij} = \begin{cases} 1 & i = j \\ 0 & \text{otherwise} \end{cases}$$

$$I_n$$

identity matrix

$$I = [e_1 \; e_2 \cdots e_n]$$

$$e_1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad I$$

zero matrices $\quad O_{n \times m} \rightarrow$ matrix of all 0's

$$O$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$\begin{bmatrix} I & O \\ A & I \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 2 & 3 & 1 & 0 \\ 4 & 5 & 6 & 0 & 1 \end{bmatrix}$$

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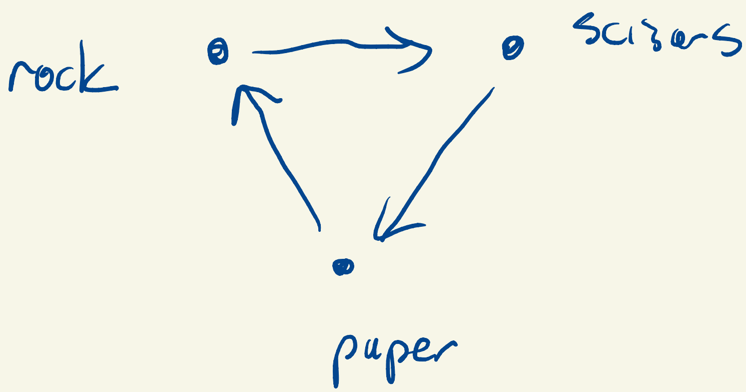
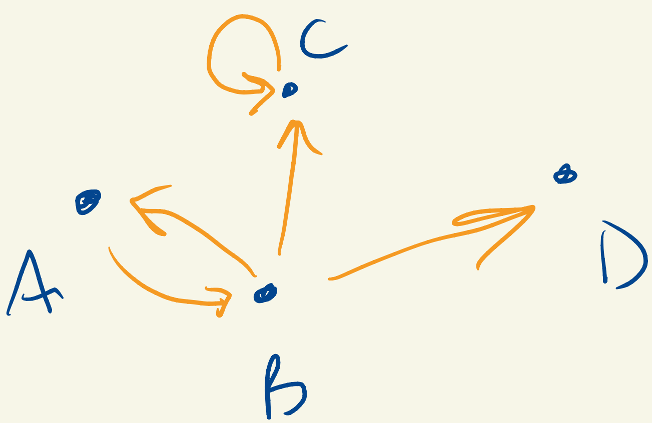
diagonal matrices $\quad \begin{bmatrix} 3 & & & O \\ & 7 & & \\ & & 1 & \\ O & & & 6 \end{bmatrix} = \mathrm{diag}(3, 7, 1, 6)$

$$\mathrm{diag}(1,1,1,1) = I_4$$

Directed Graphs



$$\begin{array}{c|ccc} & r & p & s \\ \hline r & 0 & 1 & 0 \\ p & 0 & 0 & 1 \\ s & 1 & 0 & 0 \end{array}$$



$$\begin{array}{c|cccc} & A & B & C & D \\ \hline A & 0 & 1 & 0 & 0 \\ B & 1 & 0 & 0 & 0 \\ C & 0 & 1 & 1 & 0 \\ D & 0 & 1 & 0 & 0 \end{array}$$