$$x = (1, 2, 1, -2)$$

$$y = (4, 1, 3, 2)$$

$$x^T y = 1 \cdot 4 + 2 \cdot 1 + 1 \cdot 3 - 2 \cdot 2$$

$$= 5$$

$$\|x\| = (1^2 + 2^2 + 1^2 + 2^2)^{1/2}$$

$$= \sqrt{10}$$

$$\|y\| = (4^2 + 1 + 3^2 + 2^2)^{1/2}$$

$$= \sqrt{30}$$

$$\angle(x, y)$$

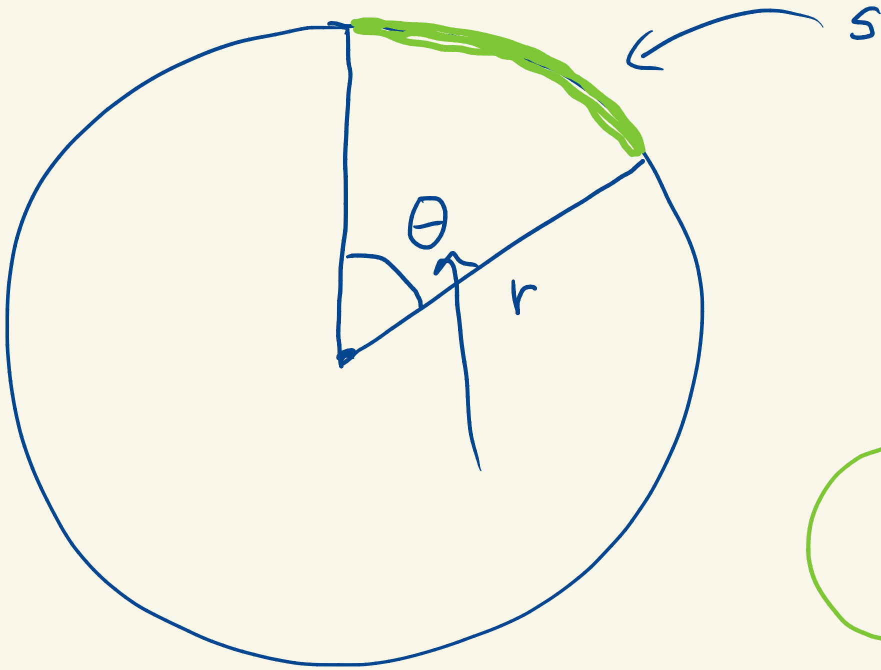
$$\underbrace{\frac{x}{\|x\|}}_{u} \qquad \underbrace{\frac{y}{\|y\|}}_{v}$$

$$\cos(\theta) = u^T v$$

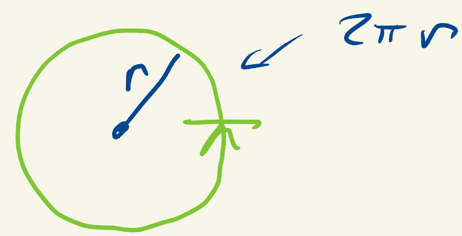
$$\cos(\theta) = \frac{x^T y}{\|x\| \, \|y\|}$$

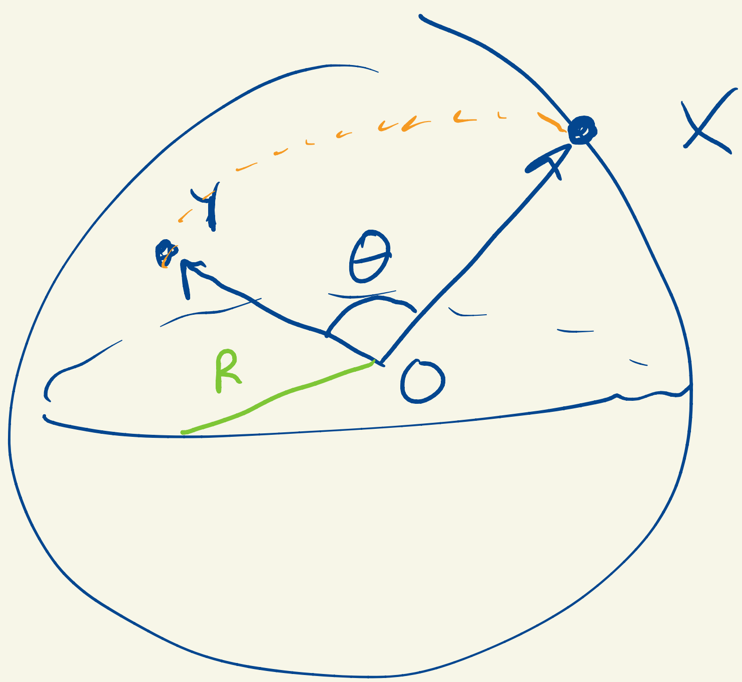
$$\cos(\theta) = \frac{5}{10^{1/2} \cdot 30^{1/2}} = \frac{1}{2\sqrt{3}}$$

$$\theta = \arccos\left(\frac{1}{2\sqrt{3}}\right) = 73^\circ$$



$s = r\theta$



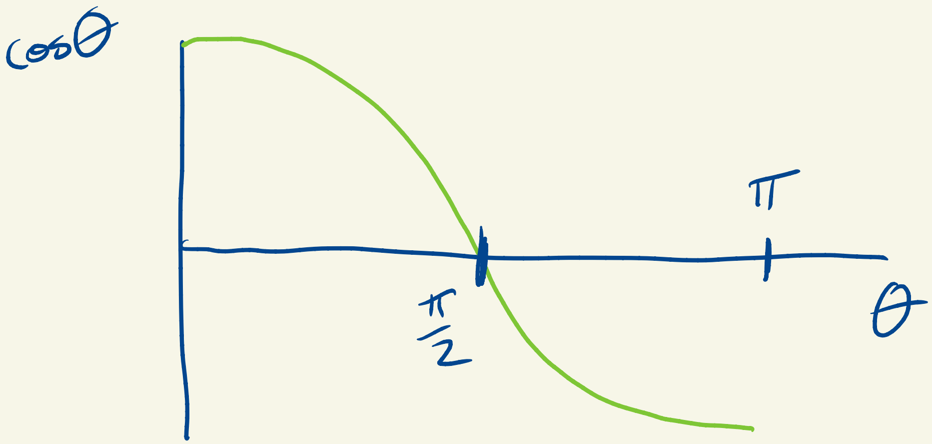


To compute distance from X to Y

1) compute $\angle(X, Y) = \theta$

2) $s = R\theta$

↳ in radians



Consequences of the sign of $x^T y$

$$x^T y = \|x\| \, \|y\| \cos(\theta)$$

$$x^T x = x_1 x_1 + x_2 x_2 \ldots$$
$$= \|x\|^2$$

If $x^T y = 0$ then

$\cos\theta = 0$ so $\theta = \frac{\pi}{2}$ rad
$= 90°$

so x and y are perpendicular

we write $x \perp y$ orthogonal

perpendicular

If $x^T y > 0$ then $\angle(x,y)$ is acute

If $x^T y < 0$ then $\angle(x,y)$ is obtuse

$$|x^T y| \leq \|x\| \, \|y\| \leftarrow$$ Cauchy-Schwartz inequality

(with equality if and only if x and y are multiples of each other. (i.e. parallel)

$y = 3x$

$x^T y = 3\|y\|^2$

If $x = 0$ or $y = 0$ the inequality is obvious. (it's an equality!)

numbers

$$0 \le \|\alpha x - \beta y\|^2 = (\alpha x - \beta y)^T (\alpha x - \beta y)$$

$$= \alpha^2 x^T x - \beta \alpha y^T x - \alpha \beta x^T y + \beta^2 y^T y$$

$\alpha x - \beta y = 0$

$\alpha x = \beta y$

$$= \alpha^2 \|x\|^2 - 2\alpha\beta x^T y + \beta^2 \|y\|^2$$

Clever step: let's set $\alpha = \|y\|$ and $\beta = \|x\|$.

$$0 \le \|y\|^2 \|x\|^2 - 2\|x\|\|y\| x^T y + \|x\|^2 \|y\|^2$$

$$2\|x\|\|y\| x^T y \le 2\|x\|^2\|y\|^2$$

$$x^T y \le \|x\|\|y\|$$

$-y$ $\qquad\qquad$ $-7 \leq z \leq 7$

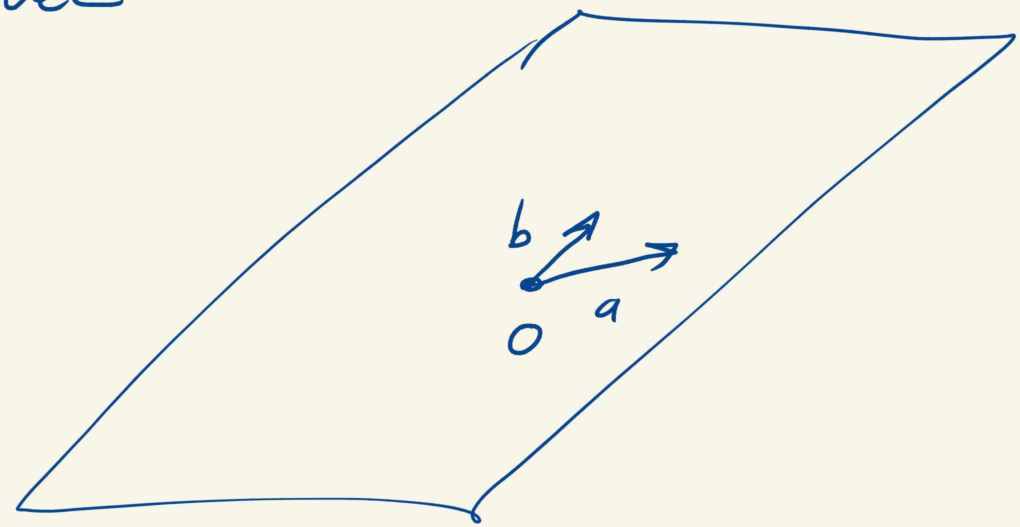
$$x^T(-y) \leq \|x\| \, \|-y\|$$

$$-x^T y \leq \|x\| \, \|y\|$$

$$-\|x\| \, \|y\| \leq x^T y \leq \|x\| \, \|y\|$$

$$|x^T y| \leq \|x\| \, \|y\|$$

Ch 5 Linear Independence



$$\alpha a + \beta b = c$$

$\alpha = 1$

$\alpha = 1/2$