Defined locally Euclidean spaces of dimension n.

each $p \in M$ has a nbhd U homeomorphic to

- $\mathbb{R}^n$
- $B_r(0) \subseteq \mathbb{R}^n$
- $\boxed{\text{an open set in } \mathbb{R}^n}$

Def: A manifold of dimension n is a topological space that is

1) locally Euclidean of dimension n
2) Hausdorff
3) 2^{nd} countable

You can have 1) but any combination or not of

2) and 3)

A locally Euclidean space satisfies 2) + 3) $\Leftrightarrow$

it metrizable + separable.

Examples: • $\mathbb{R}^n$

• If $U \subseteq \mathbb{R}^n$ is open then U is an n-manifold

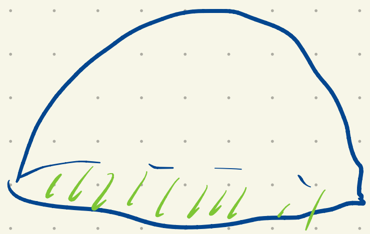
1), 2), $\underbrace{3)}$, all easy

→ last class

• Spaces that are homeomorphic to open subsets of some $\mathbb{R}^n$?

$$M = \{(x,y,z) \in \mathbb{R}^3 : x^2+y^2<1,\ z = \sqrt{1-x^2-y^2}\}$$

$$U = \{(x,y) \in \mathbb{R}^2 : x^2+y^2<1\}$$



$$\pi : M \to U$$

$$\pi(x,y,z) = (x,y)$$

Maps into $\mathbb{R}^n$ are continuous iff the component maps are continuous.

$$(x,y,z) \to x$$
$$(x,y,z) \to y$$

Sequences in $\mathbb{R}^n$ converge iff each component converges individually.

Restrictions of continuous functions to subsets are continuous.

Restriction of codomain does not affect continuity.

$\pi^{-1}(x,y) = (x, y, \sqrt{1-x^2-y^2})$ is continuous.

4) Locally Euclidean at dimension 0 $\Leftarrow\Rightarrow$ discrete

$\Rightarrow$ Hausdorff

2nd countable $\Leftarrow\Rightarrow$ countable.

0-manifolds are countable discrete spaces

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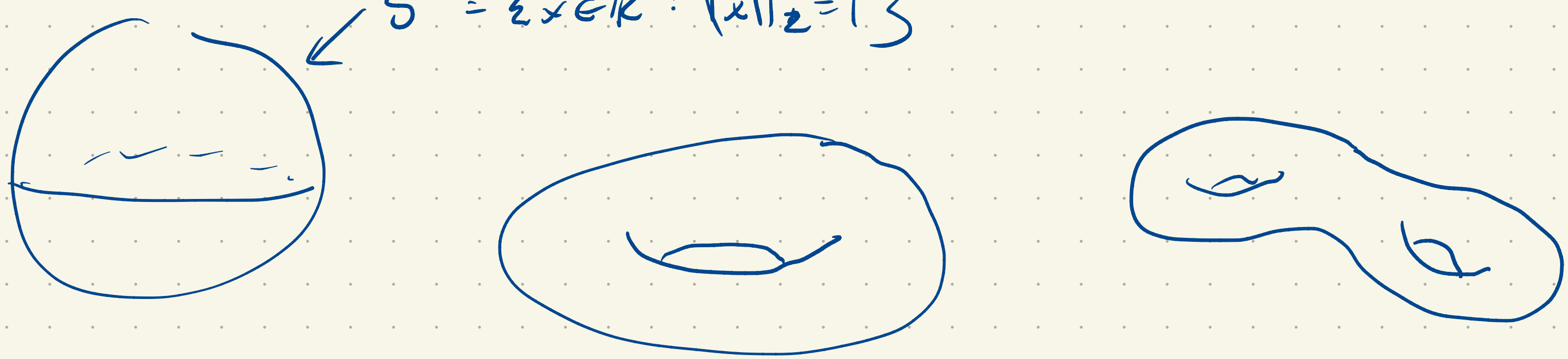
5) $S^2 = \{x \in \mathbb{R}^3 : \|x\|_2 = 1\}$

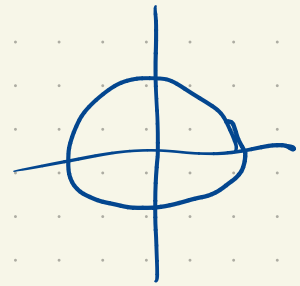


6) $S^n = \{x \in \mathbb{R}^{n+1} : d_2(x,0) = 1\}$ spheres

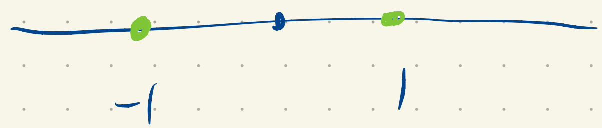
each is an n-manifold.

S^2

S^1



S^0

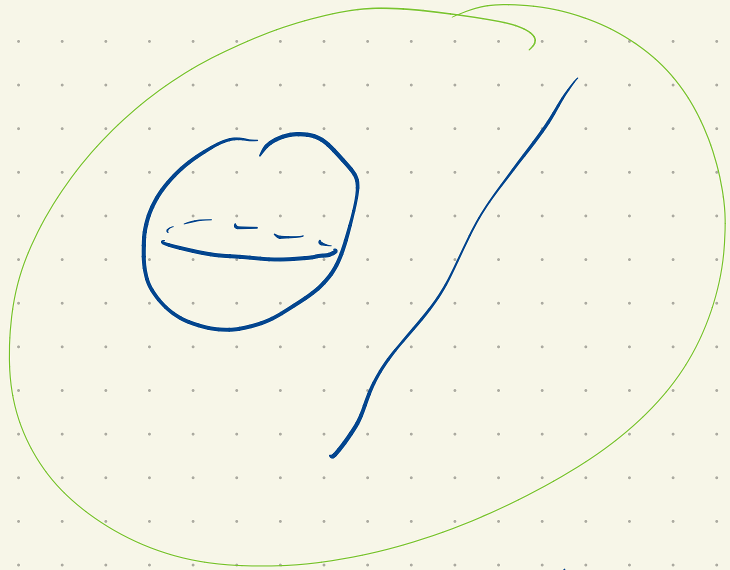


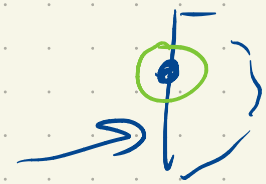
7) Real projective space $\mathbb{RP}^n$

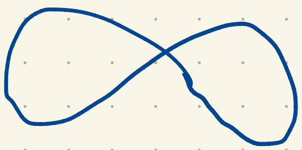
is sets the points in $\mathbb{RP}^n$ are the lines through the origin in $\mathbb{R}^{n+1}$

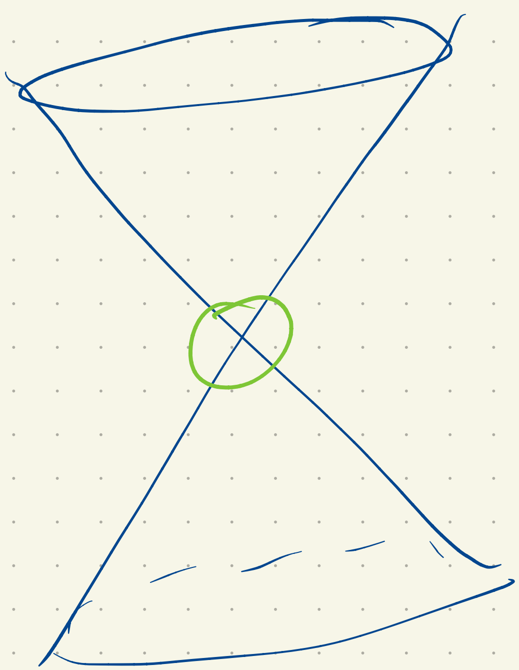
These are n-manifolds

Non-manifolds



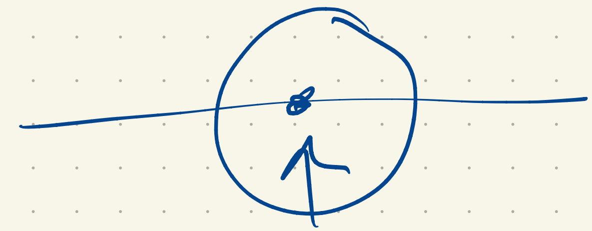






2-manifold

A manifold with boundary.



$$\mathbb{H}^n = \{ (x_1, \dots, x_n) \in \mathbb{R}^n : \ x_n \geq 0 \}$$

upper half space

An n-manifold with boundary is a top space that is Hausdorff, 2nd countable such that each point admits a neighbourhood homeomorphic to an open subset of $\mathbb{H}^n$.

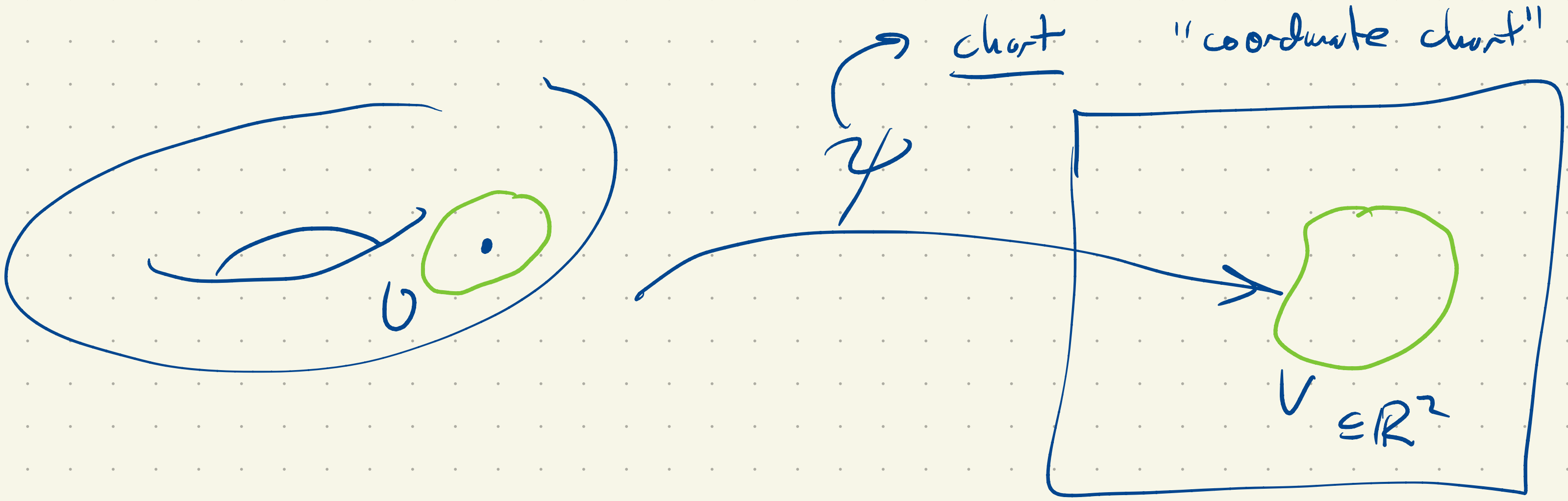
Hmk:- A manifold with boundary need not be a manifold.

Chapter 13 contains the theory need to show

the fact above as well as

the fact that no manifold can have more than one dimension.

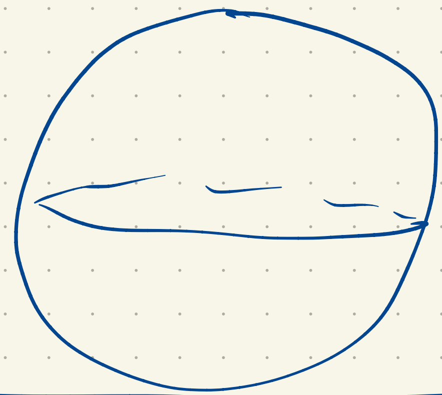
We'll see this for 0, 1 and 2-manifolds.



$\psi: U \to V$ is a homeomorphism

The components of φ are called coordinates

$$\varphi(p) = (\varphi_1(p), \varphi_2(p))$$



Chapter 3 New spaces from old.

Given a subset $A \subseteq X$ we'll put a natural topology on A.