Note: The book has Exercises, which are interspersed among the prose, and Problems, which appear at the ends of the chapters. It can be easy to confuse the two. Exercises are denoted in blue.

1. Problem 4-11 b) (Just the connected part; the path connected is similar)

2. Problem 4-9 [Modified] Let M be an n-manifold.

a) Show that each component of M is a (connected) manifold.

b) Show that there are at most countably many components.

c) Suppose $f : M \to Z$ is a map into a topoogical space Z. Show that f is continuous if and only if its restriction to each component is.

d) Read Theorem 3.41. Then conclude that an n-manifold is homeomorphic to a disjoint union of countably many connected n-manifolds.

3. Let $f : X \to Y$ where X is compact and Y is Hausdorff. Show that f is continuous if and only if the graph of f is closed in $X \times Y$. The graph of f is $G_f = \{(x, f(x)) : x \in X\}$.

4. If (X, d) is a metric space, a function $f : X \to X$ is an isometry if for all $x, y \in X$, $d(f(x), f(y)) = d(x, y)$. Show that every isometry is continuous and injective. Then show that if X is compact and f is an isometry then f is surjective as well and quickly conclude that f is a homeomorphism. Hint: Show that a is not in the image of f, then for some $\epsilon > 0$, $B_\epsilon(a)$ is also not in the image of f. Then show that if $x_0 = a$, $x_1 = f(x_0)$, etc, then $d(x_n, x_m) > \epsilon$ for $n \neq m$.

5. Show that if p and q are elements of the interior of the closed unit ball

$$\mathbb{B}^n = \{x \in \mathbb{R}^n : |x| \le 1\},$$

then there is a homeomorphism $\phi : \mathbb{B}^n \to \mathbb{B}^n$ such that $\phi(p) = q$ and such that $\phi(x) = x$ for all x with $|x| = 1$. Be as rigorous as you can, but avoid writing a tome.

6. 3.22a You'll need to read the material on pages 78–80 first.