Note: The book has Exercises, which are interspersed among the prose, and Problems, which appear at the ends of the chapters. It can be easy to confuse the two. Exercises are denoted in blue.

1. Suppose $\mathcal{B}_1$ and $\mathcal{B}_2$ are bases for topologies τ_1 and τ_2. Show that $\tau_1 \subseteq \tau_2$ if and only if for every $B_1 \in \mathcal{B}_1$ and every $x \in B_1$ there is a $B_2 \in \mathcal{B}_2$ such that $x \in B_2 \subseteq B_1$.

2. Given a family $\{\tau_\alpha\}_{\alpha \in I}$ of topologies in X, show that there is a unique smallest topology containing each τ_α. Show also that there is a unique largest topology contained in each τ_α. Take advantage of past work!

3. Let $\mathcal{B} = \{[a,b) : a, b \in \mathbb{Q}\}$. Show that $\mathcal{B}$ is a pre-basis and hence generates a topology $\tau_{\mathcal{B}}$. Compare this topology to the lower-limit topology τ_ℓ. In particular, determine if it is finer or coarser or neither or both.

4. Problem 2-12

Conclude that the topology generated by a pre-basis $\mathcal{B}$ is the smallest topology in which every set from $\mathcal{B}$ is open.

5. Problem 2-15

6. Let A be a subset of a topological space X, and let $\mathcal{B}$ be a basis for the topology.

a) Show that $x \in \overline{A}$ if and only if for every $B \in \mathcal{B}$ with $x \in B$, $B \cap A \neq \varnothing$.

b) Show that $x \in \partial A$ if and only if for every $B \in \mathcal{B}$ with $x \in B$, $B \cap A \neq \varnothing$ and $B \cap A^c \neq \varnothing$

c) Show that $\mathrm{Int}(A) \cap \partial A = \varnothing$ and $\overline{A} = \mathrm{Int}(A) \cup \partial A$.

You are not allowed to cite Proposition 2.8 to prove these results.

7. Problem 2-20