In the problem below you can use the following fact. If α and β are complex numbers, not both zero, and if x and y are complex numbers such that

$$\alpha x + \beta y = 0$$

then there exists a complex number z such that $(x, y) = z(-\beta, \alpha)$.

1. Suppose $A \in SU(2)$. That is, A is a 2×2 complex matrix with $A\bar{A}^T = I$ and with $\det(A) = 1$. Show that there exist complex numbers α and β such that

$$A = \begin{pmatrix} \alpha & -\beta \\ \overline{\beta} & \overline{\alpha} \end{pmatrix}$$

and such that $|\alpha|^2 + |\beta|^2 = 1$.

Hint: without loss of generality you can assume A has the form

$$A = \begin{pmatrix} \alpha & -\beta \\ \overline{\gamma} & \overline{\delta} \end{pmatrix}$$

for some complex numbers α, β, γ, δ. Now use the condition $A\overline{A}^T = I$.