1. Suppose $T : \mathbb{R}^n \to \mathbb{R}^n$ is a linear map and $|T(u)| = |u|$ for all u in $\mathbb{R}^n$. Show that T is an isometry.

2. Show that for all $u, v \in \mathbb{R}^n$,

$$u \cdot v = \frac{1}{2}\left(|u - v|^2 - |u|^2 - |v|^2\right).$$

Thus, if you can compute distances from 0, then you can also compute dot products!

3. Suppose $T : \mathbb{R}^n \to \mathbb{R}^n$ is a linear map and $|T(u)| = |u|$ for all u in $\mathbb{R}^n$. Show that T preserves the dot product, i.e. that for all $u, v \in \mathbb{R}^n$

$$u \cdot v = T(u) \cdot T(v).$$

Hint: look at the previous problem.

4. Suppose $T : \mathbb{R}^n \to \mathbb{R}^n$ is linear and that

$$T(e_i) \cdot T(e_j) = e_i \cdot e_j$$

for some basis vectors $e_1, \ldots, e_n$ in $\mathbb{R}^n$. Show that $T(u) \cdot T(v) = u \cdot v$ for all $u, v \in \mathbb{R}^n$.

5. 3.1.2