1. 2.2.4

2. Suppose $T : \mathbb{R} \to \mathbb{R}$ is an isometry and that $T(0) = 0$. Give a direct proof that either $T(x) = x$ for all x or $T(x) = -x$ for all x.

3. Let u be a nonzero vector in $\mathbb{R}^n$. Show that for every $x \in \mathbb{R}^n$ there is a unique number a and unique vector w perpendicular to u such that $x = au + w$. Hint: take a dot product to determine what a must be.

4. Recall that a reflection through a hyperplane through the origin in $\mathbb{R}^n$ is a linear map $T : \mathbb{R}^n \to \mathbb{R}^n$ such that there exists a nonzero vector u with

1. $T(u) = -u$
2. $T(w) = w$ whenever w is perpendicular to u.

Show that such a map is an isometry.

5. Suppose $T : \mathbb{R}^n \to \mathbb{R}^n$ is an isometry and that $T(0) = 0$. Suppose moreover that $T(v) = w$ and that $w \neq v$. Show that the midpoint $(v+w)/2$ is perpendicular to the vector $u = v-w$. Also, draw a diagram that contains 0, v, w, u and $(v + w)/2$.