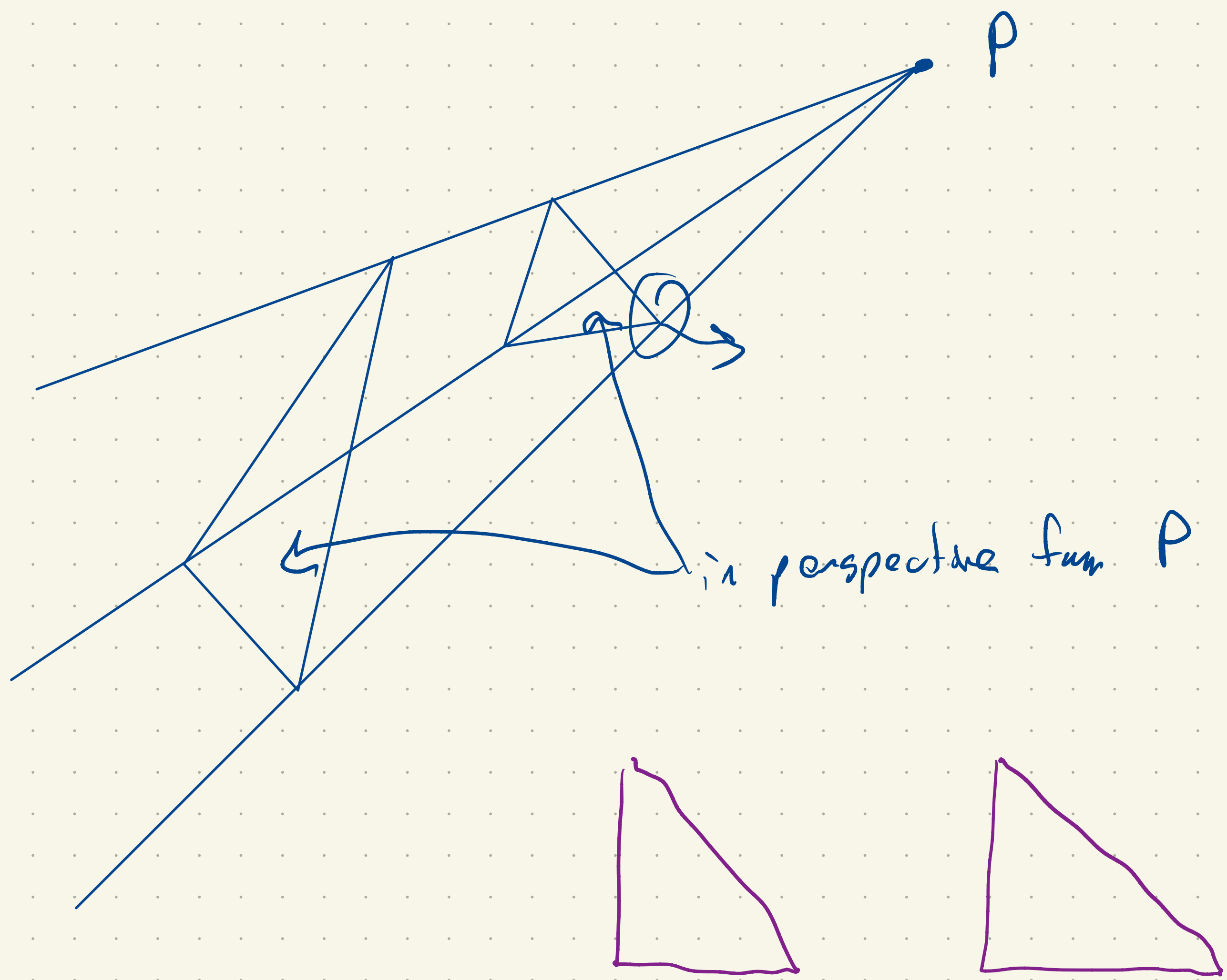
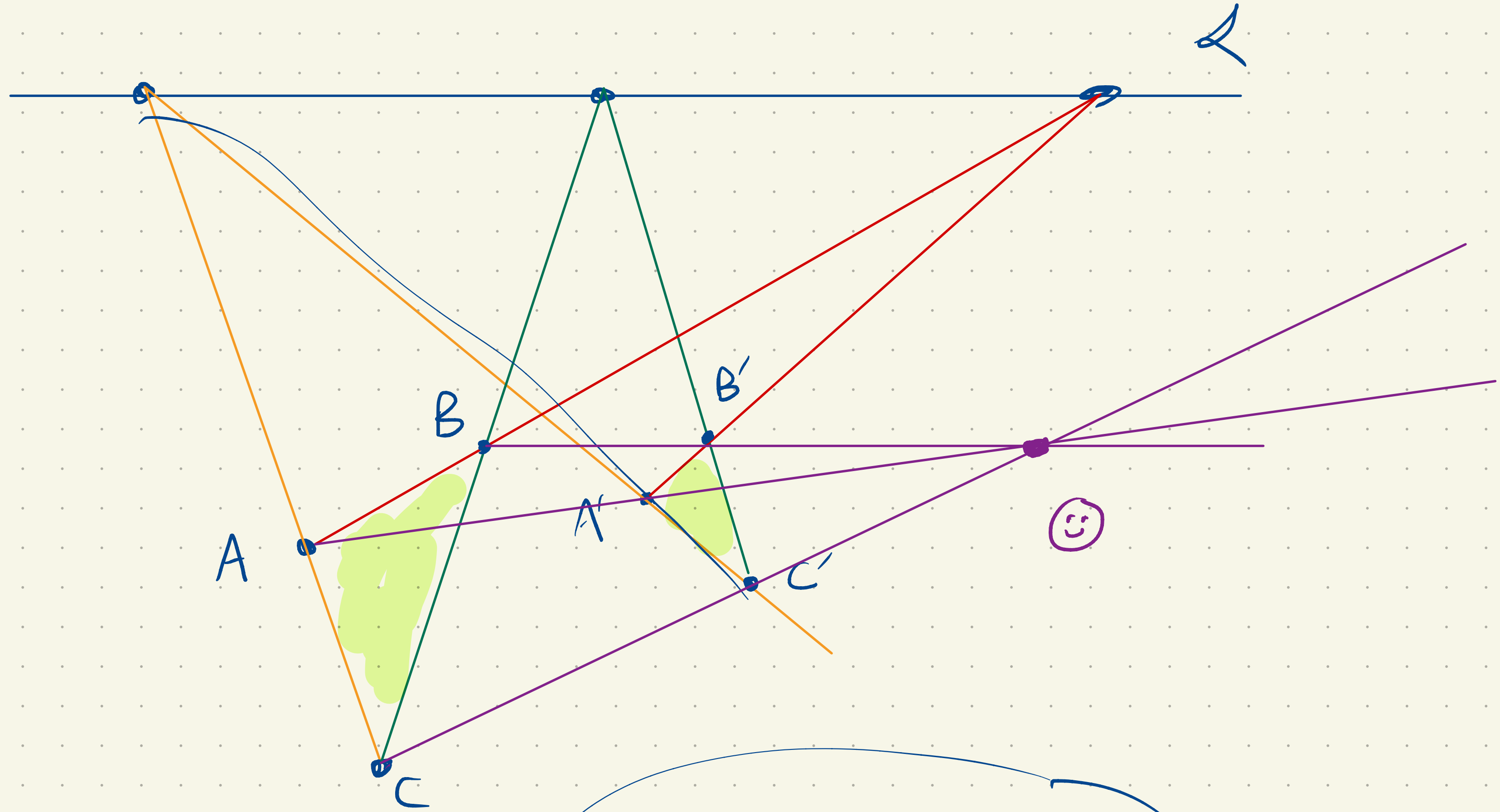
Desargues' Thm



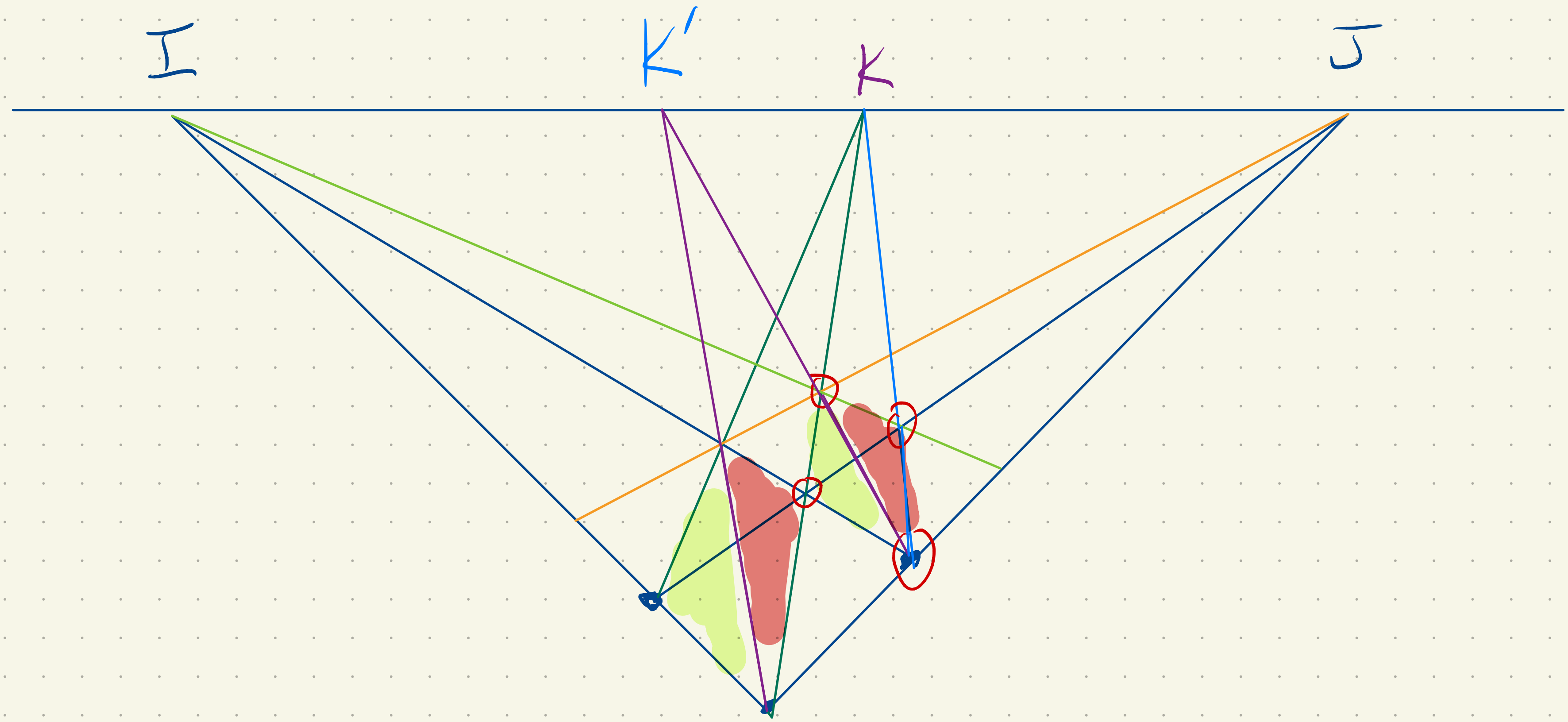


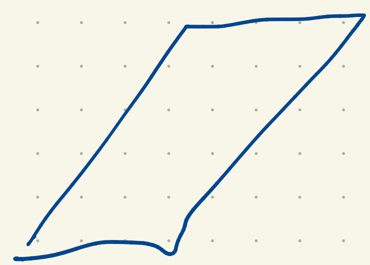
Two triangles in perspective from a line

Desargues Thm Pf A

If two triangles are in persp from a point they they are in perspective from a line.

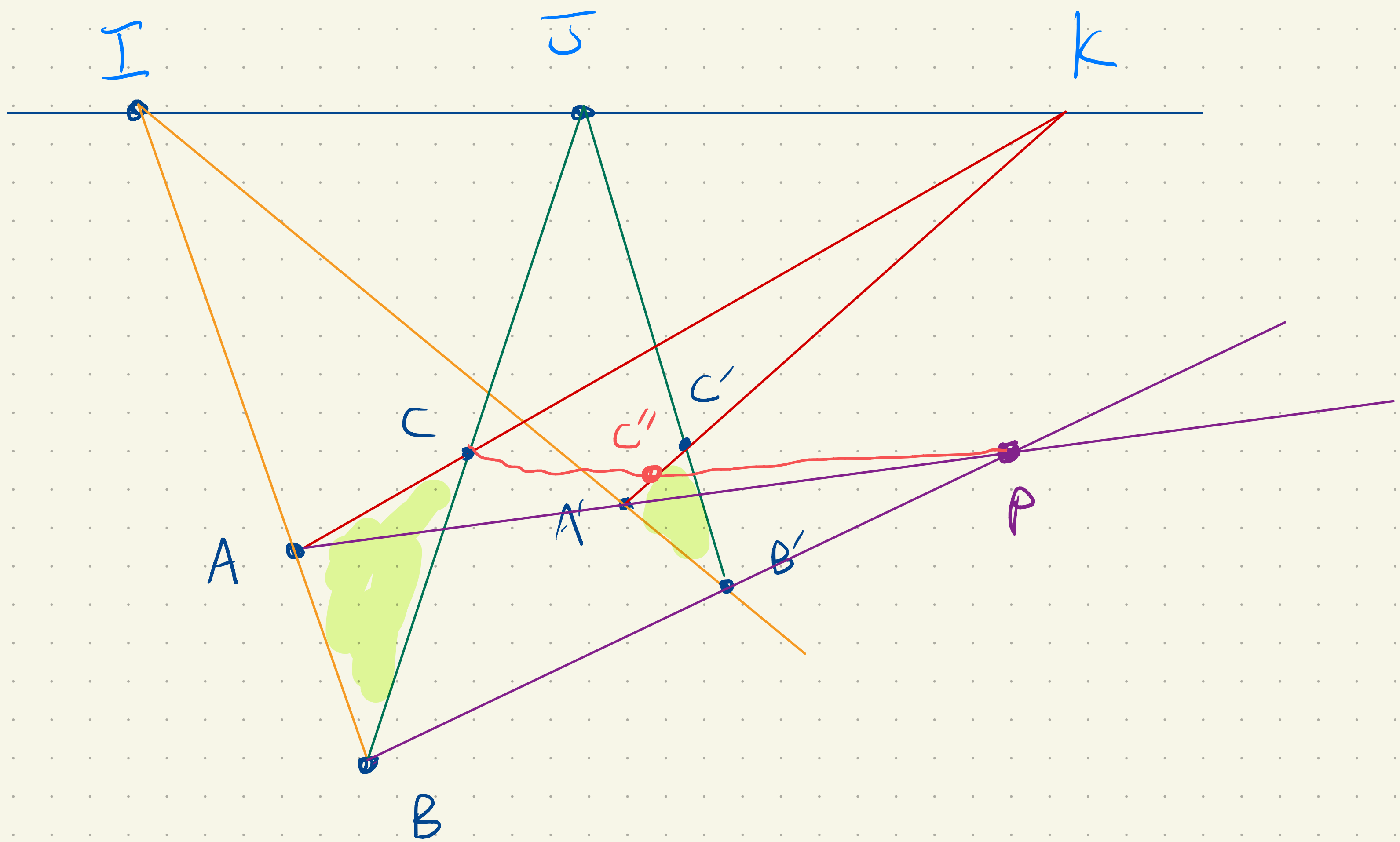
Pf B: Converse!


I
K'
K
J



If two lines are in perspective from a point, then they are in persp. from a line.

Want to show the converse holds.



We form CP and let $C'' = CP \cap A'K$

Goal: $C' = C''$!

Triangles ABC, $A'B'C''$ are in persp. from P.

This line contains $AB \cap A'B' = I$

$AC \cap A'C' = K$

So the triangles are in persp from IK,

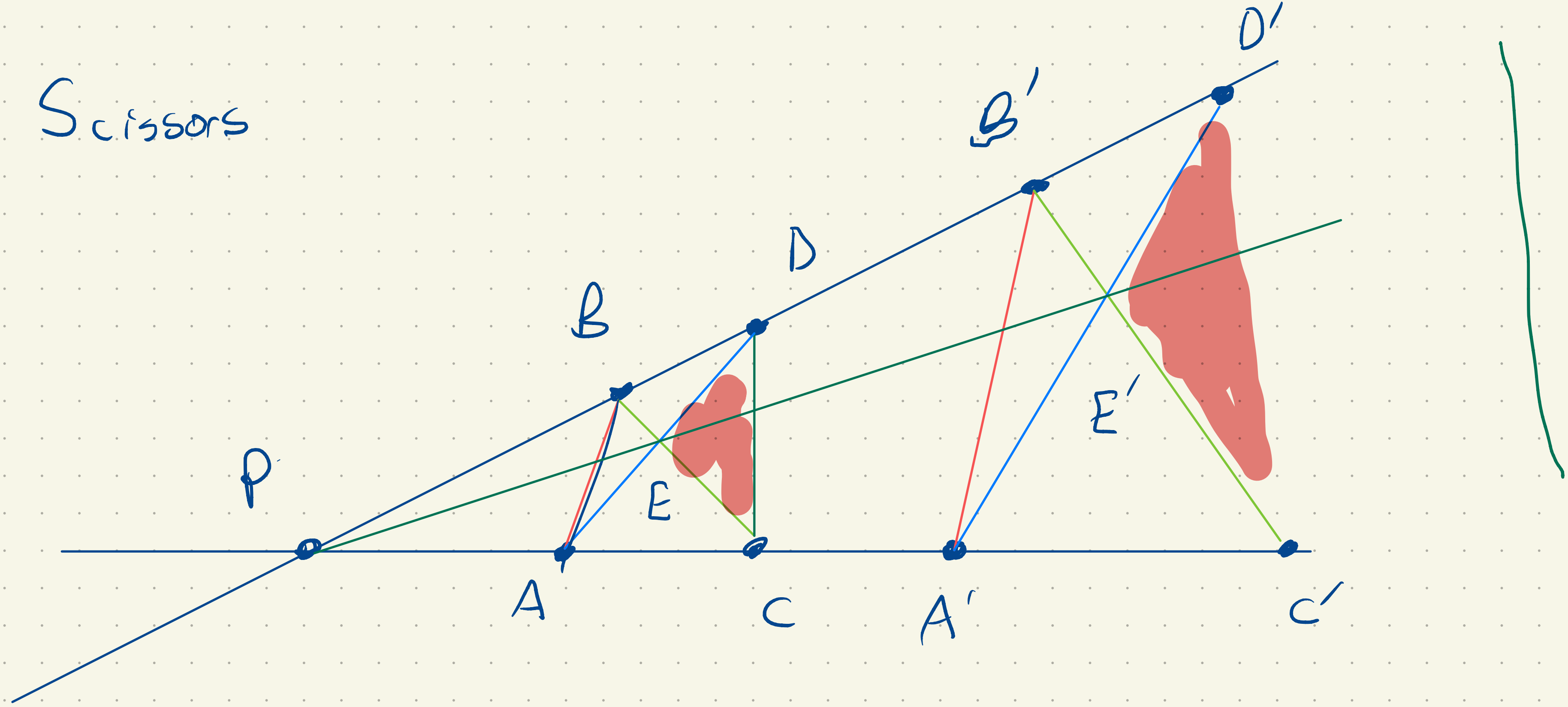
So BC and $B'C''$ intersect at J, so C''

lies on $B'J$. But C'' lies on $A'K$

by construction. The point of intersection is C'.

So $C'' = C'$.

Scissors



If $AB \cap A'B'$, $BC \cap B'C'$, $AD \cap A'D$ lie on a common line (rare!)

then $CD \cap C'D'$ lies on the same line.

Pf: Let $E = BC \cap AD$ and $E' = B'C' \cap A'D'$

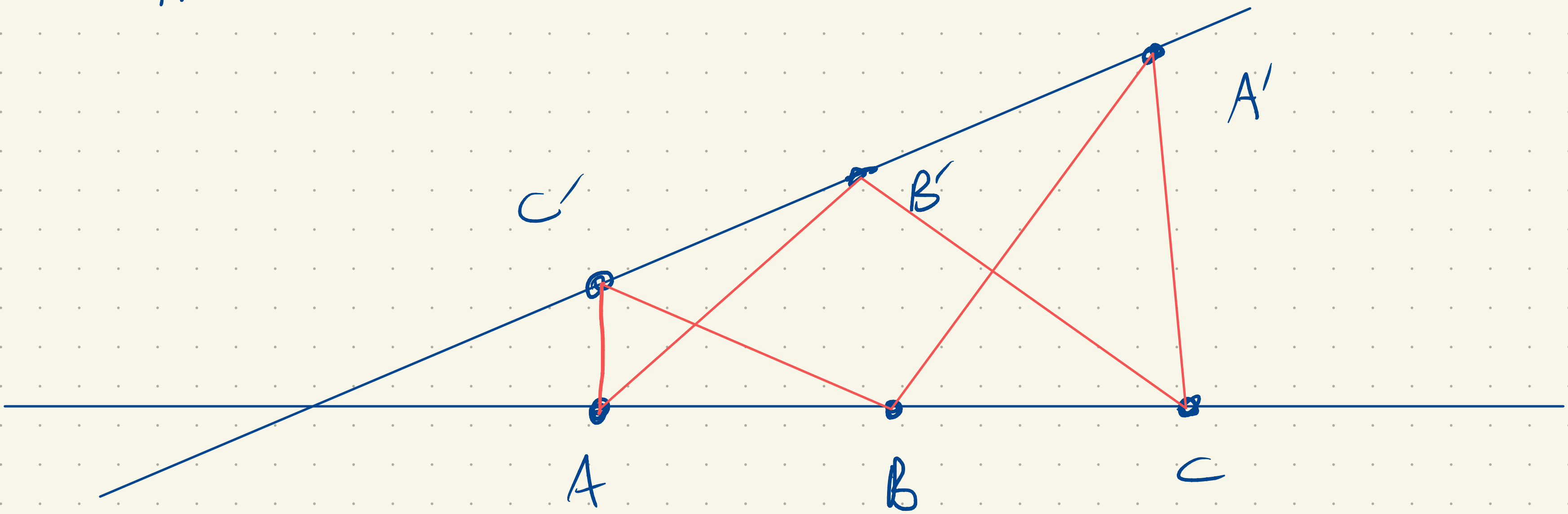
Then ΔABE and $\Delta A'B'E'$ are in persp. from a line and hence also a point which must be P.

So the line PE contains E'.

Now DEC and $D'E'C'$ are in persp. from P and hence from some line. So $CD \cap C'D'$ lies on the line determined by $AD \cap A'D'$
$CB \cap C'B'$

which is the original line.

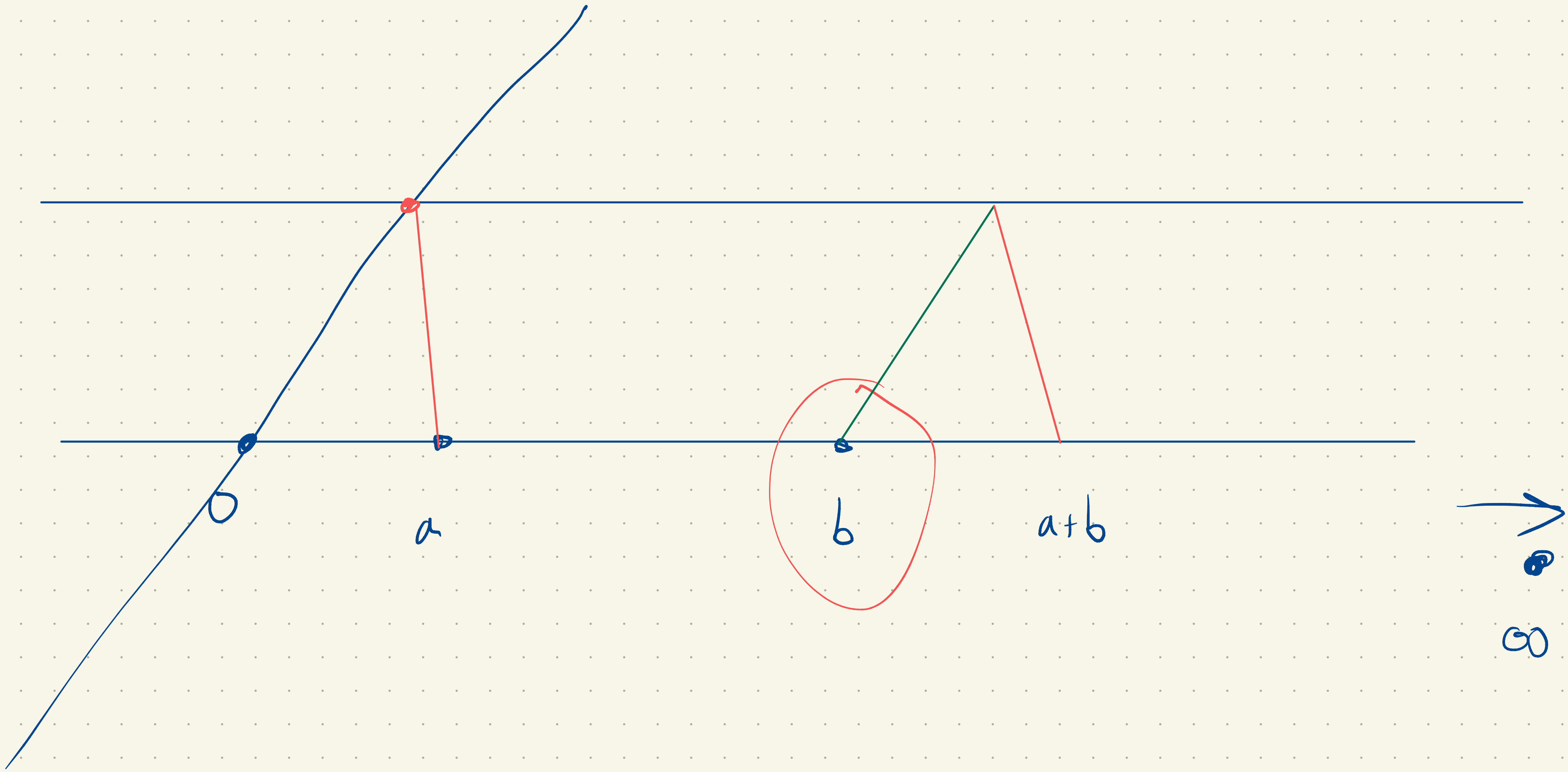
Pappus' Thm:



Hexagon $\underbrace{AB'CA'BC'}_{\text{alternate vertices lie on two lines}}$

conclusion

$$\left.\begin{array}{l} AB' \cap A'B \\ BC' \cap B'C \\ AC' \cap A'C \end{array}\right\} \text{ lie on a common line.}$$


0
a
b
a+b
∞