for small Δx the hyperbolic "width" is $\simeq \frac{1}{y}\Delta x$

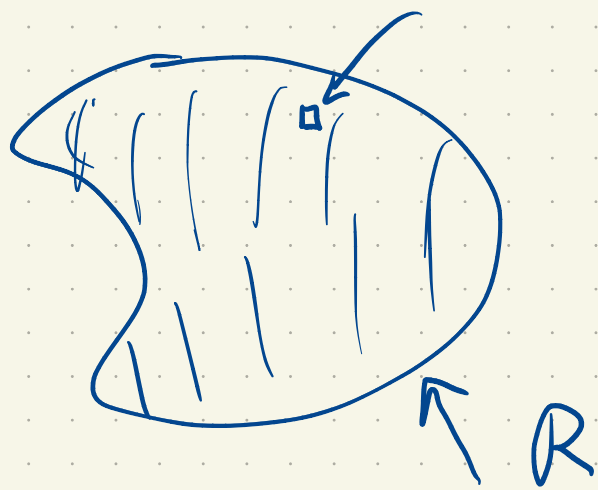
hyperbolic height $\simeq \frac{1}{y}\Delta y$

Δy (x,y)

Δx

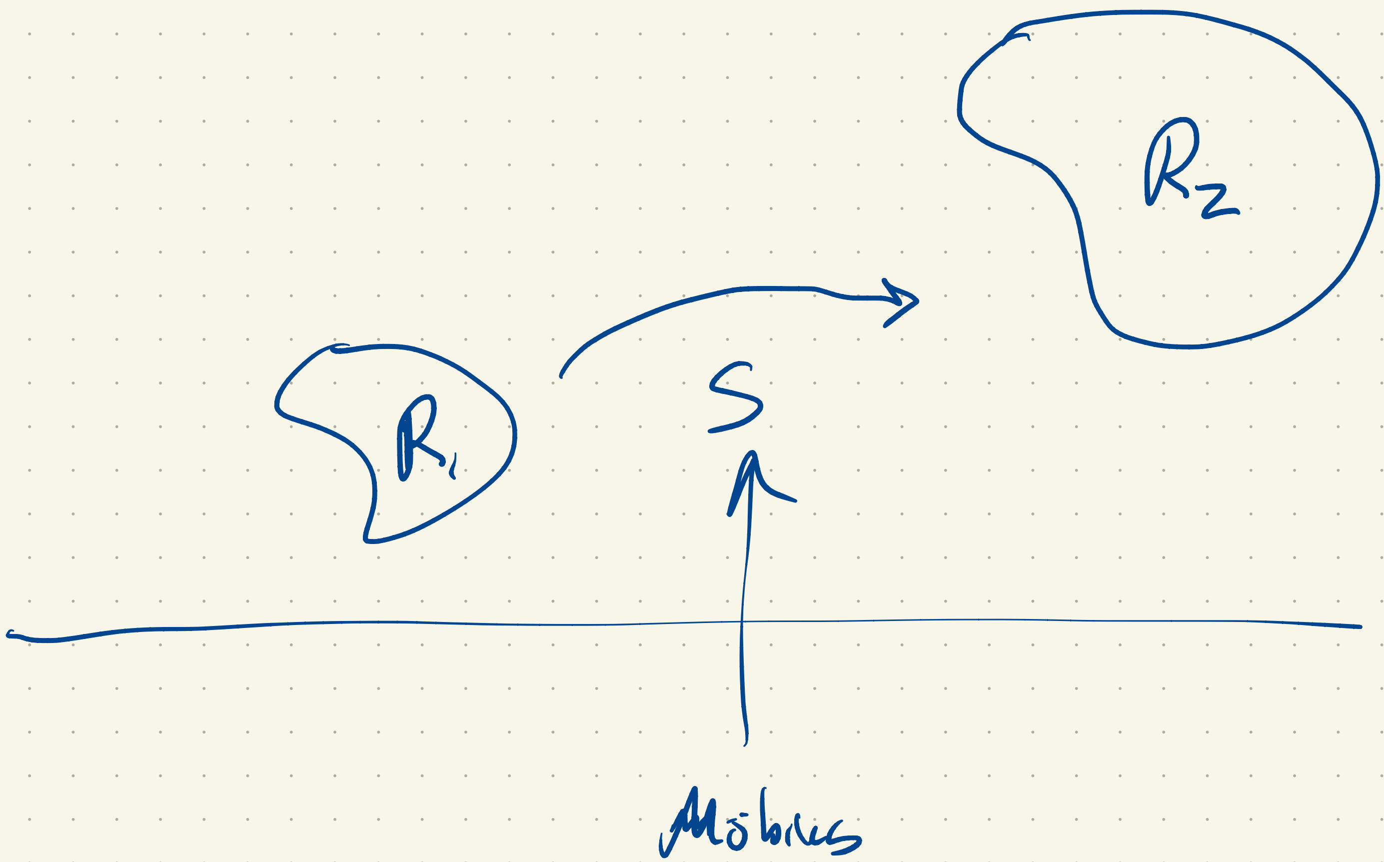
$$\int \frac{|z'(t)|}{y(t)}\,dt$$

hyperbolic area $\simeq \frac{1}{y^2}\Delta x\,\Delta y$



area of region is $\iint_R \frac{1}{y^2}\,dy\,dx$

One can show that area is an invariant.



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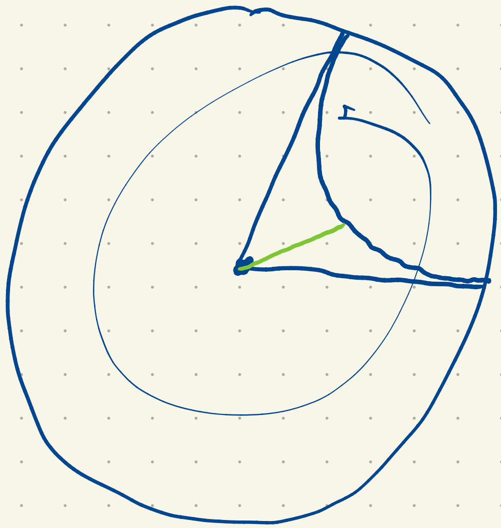

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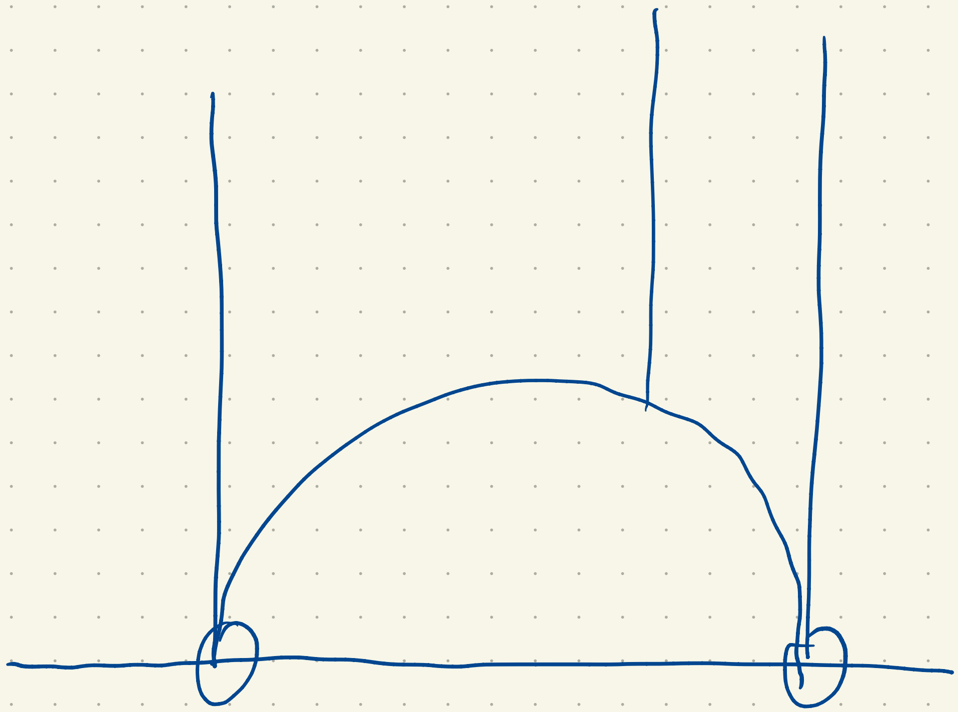
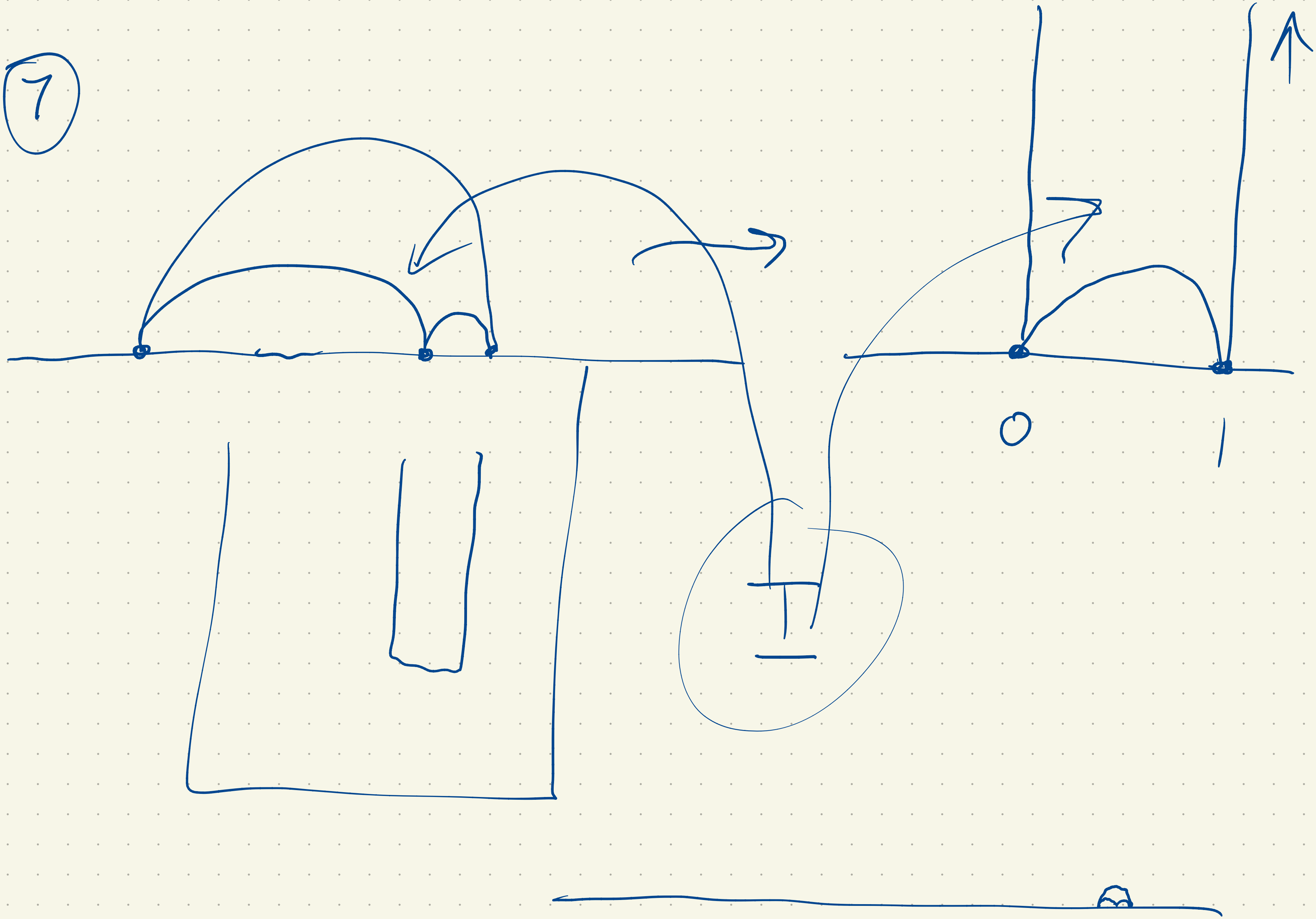
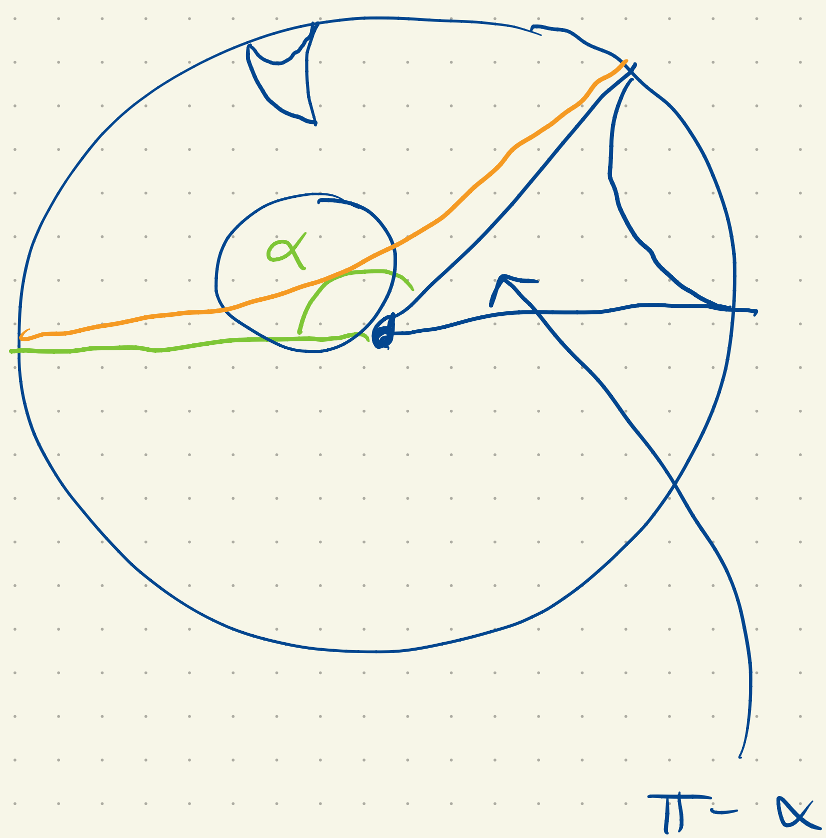


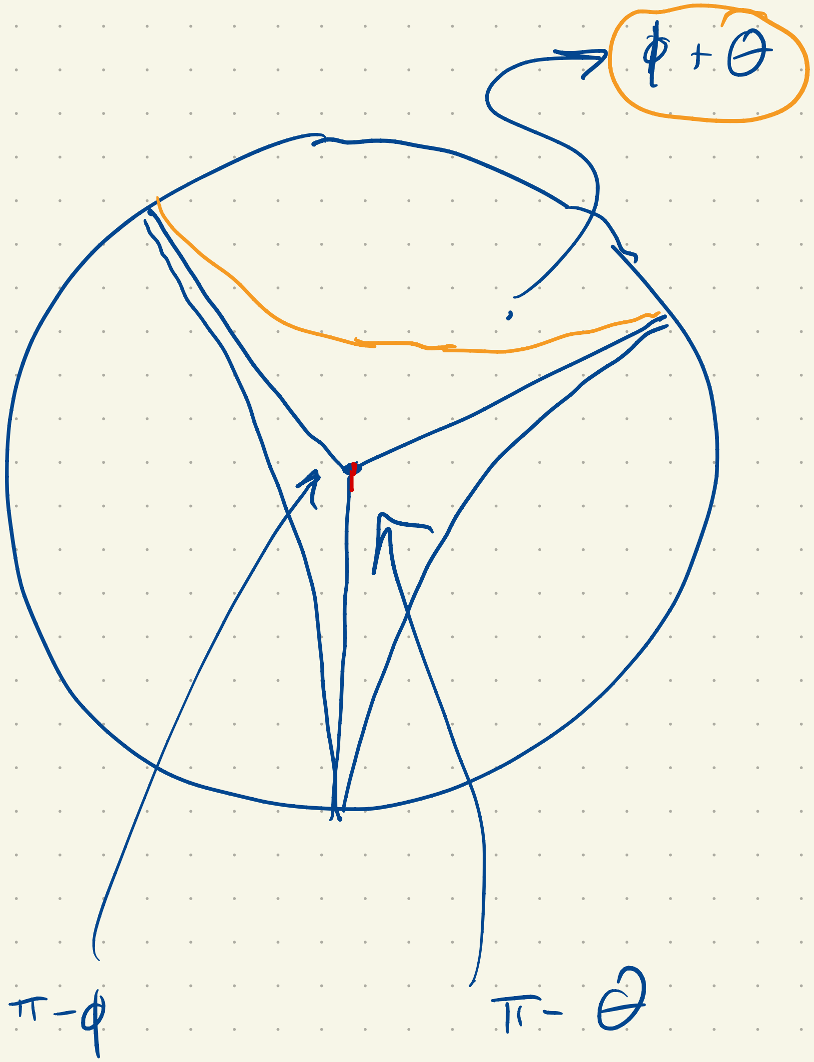

7
0
1



$T(\alpha)$ = o-ex of 2/3

w/ ext. angle α

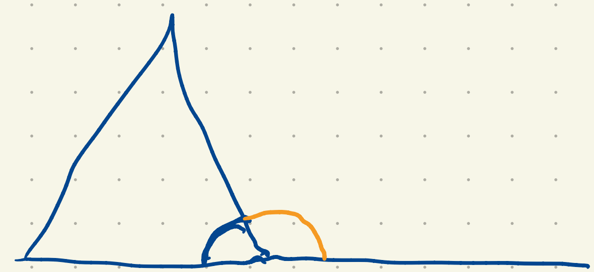
$T(\alpha) \cdot T(\pi - \alpha) = I$



$$T(\theta) + T(\phi) + T(\pi - \theta - \phi) = I$$

$$\beta + \pi - \phi + \pi - \theta = 2\pi$$

$$\beta = \theta + \phi$$



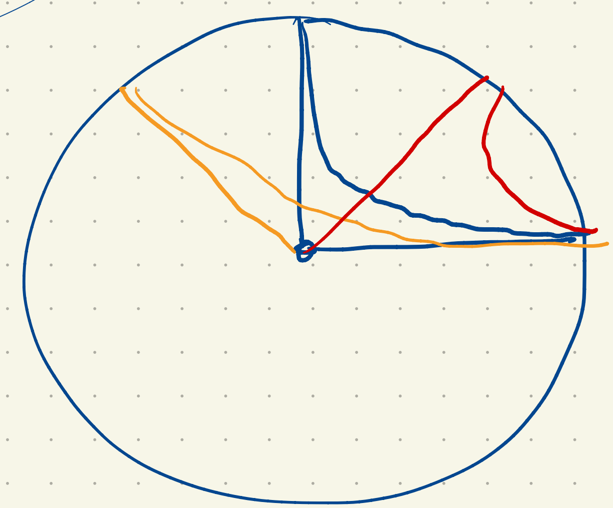
$$T(\alpha) + T(\pi - \alpha) = I \rightarrow T(\pi - \alpha) = I - T(\alpha)$$

$$T(\theta) + T(\phi) + T(\pi - \theta - \phi) = I$$

$$\downarrow$$

$$T(\theta) + T(\phi) + [I - T(\theta + \phi)] = I$$

$$T(\theta + \phi) = T(\theta) + T(\phi)$$



$$T(2\theta) = 2T(\theta)$$

$$T(\alpha) = c\,\alpha$$ for some constant c.

$$T(x+y) = T_x + T_y$$

$$T(z) = cz$$

$$T(\alpha) + T(\pi - \alpha) = I$$

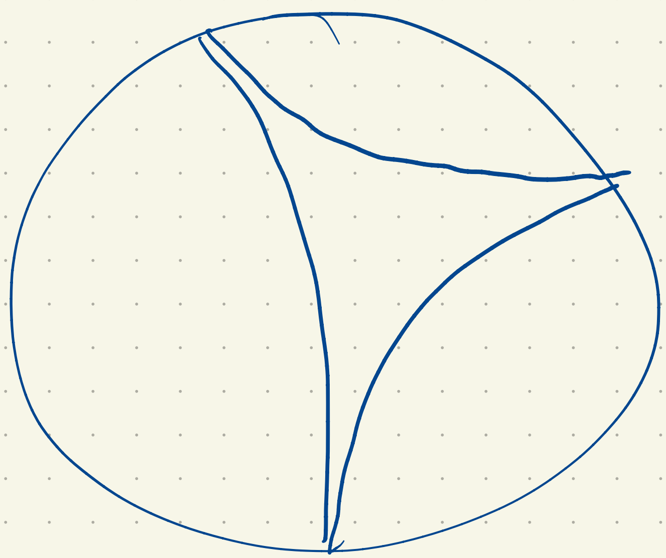
$$c\alpha + c(\pi - \alpha) = I$$

$$c\pi = I$$

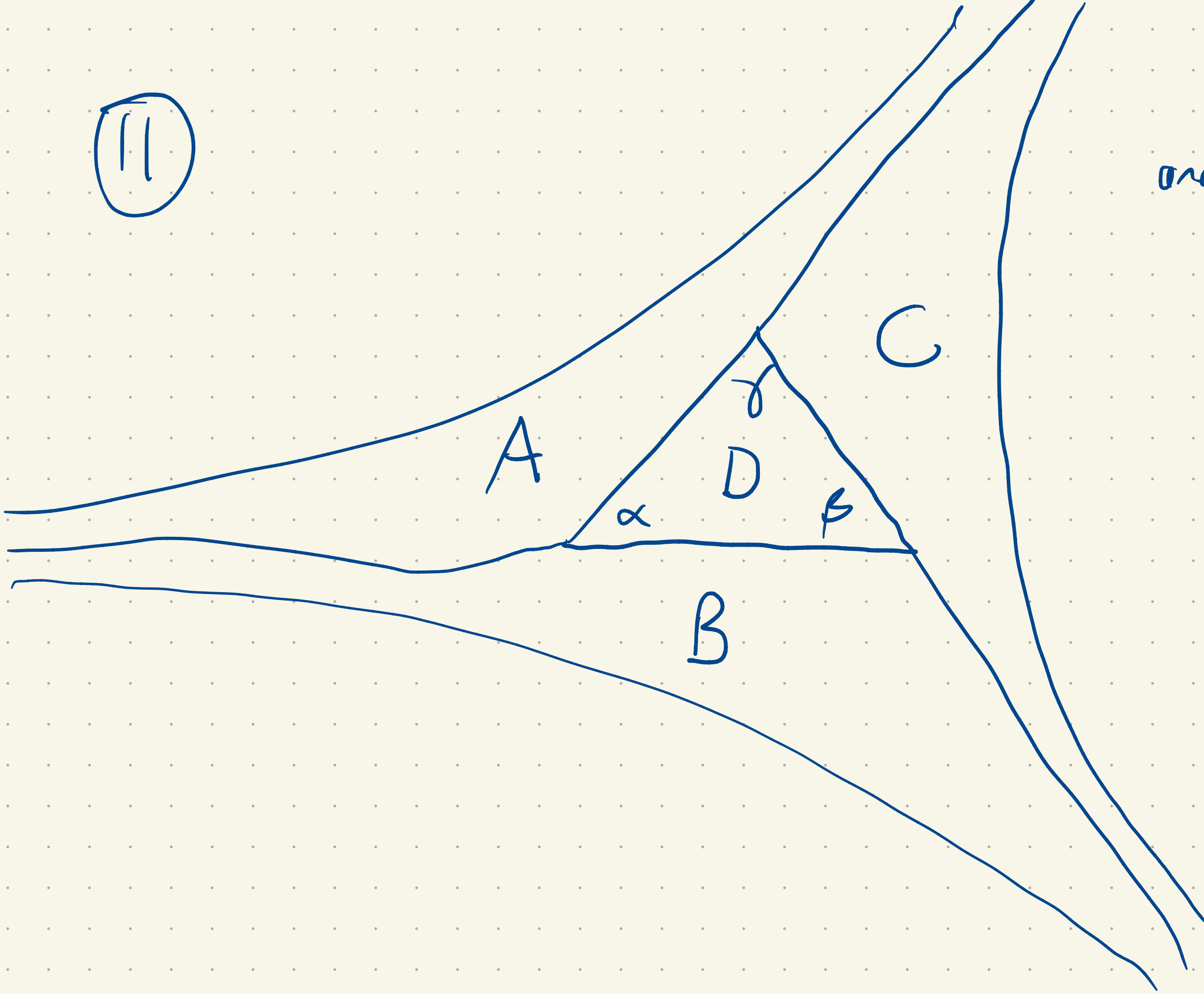
$$c = \frac{I}{\pi}$$

1D

$$T(\alpha) = \frac{I}{\pi}\alpha = I\left(\frac{\alpha}{\pi}\right)$$



11

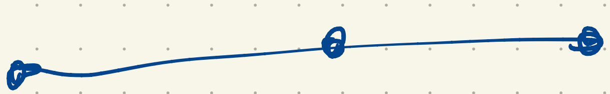


$$\text{area}(A) + \text{area}(B) + \text{area}(C) + \text{area}(D) = I$$

$$\text{area}(C) = I \frac{\gamma}{\pi}$$

$$I \frac{\alpha}{\pi} + I \frac{\beta}{\pi} + I \frac{\gamma}{\pi} + \text{area}(D) = I$$

$$\text{area}(D) = \frac{I}{\pi}\left[\pi - (\alpha + \beta + \gamma)\right]$$



angle defect of the triangle

$$\pi \geq \alpha + \beta + \gamma$$

$$\pi > \alpha + \beta + \gamma$$

$\hat{\alpha} \leftarrow$ ext. angle

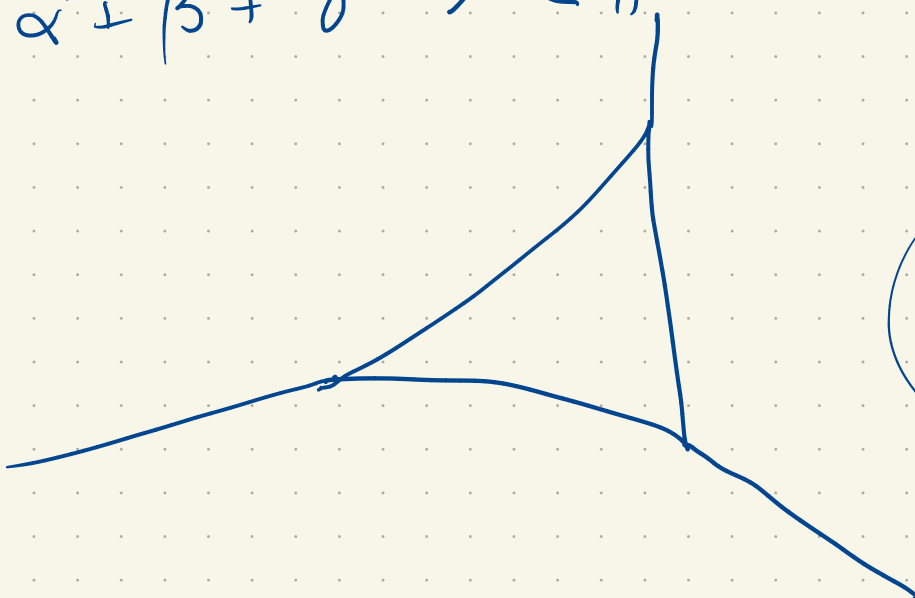
$$\pi > \pi - \hat{\alpha} + \pi - \hat{\beta} + \pi - \hat{\gamma}$$

$$\hat{\alpha} + \alpha = \pi$$

$$\hat{\alpha} = \pi - \alpha$$

$$\alpha = \pi - \hat{\alpha}$$

$$\hat{\alpha} + \hat{\beta} + \hat{\gamma} > 2\pi$$



$$I = \pi$$