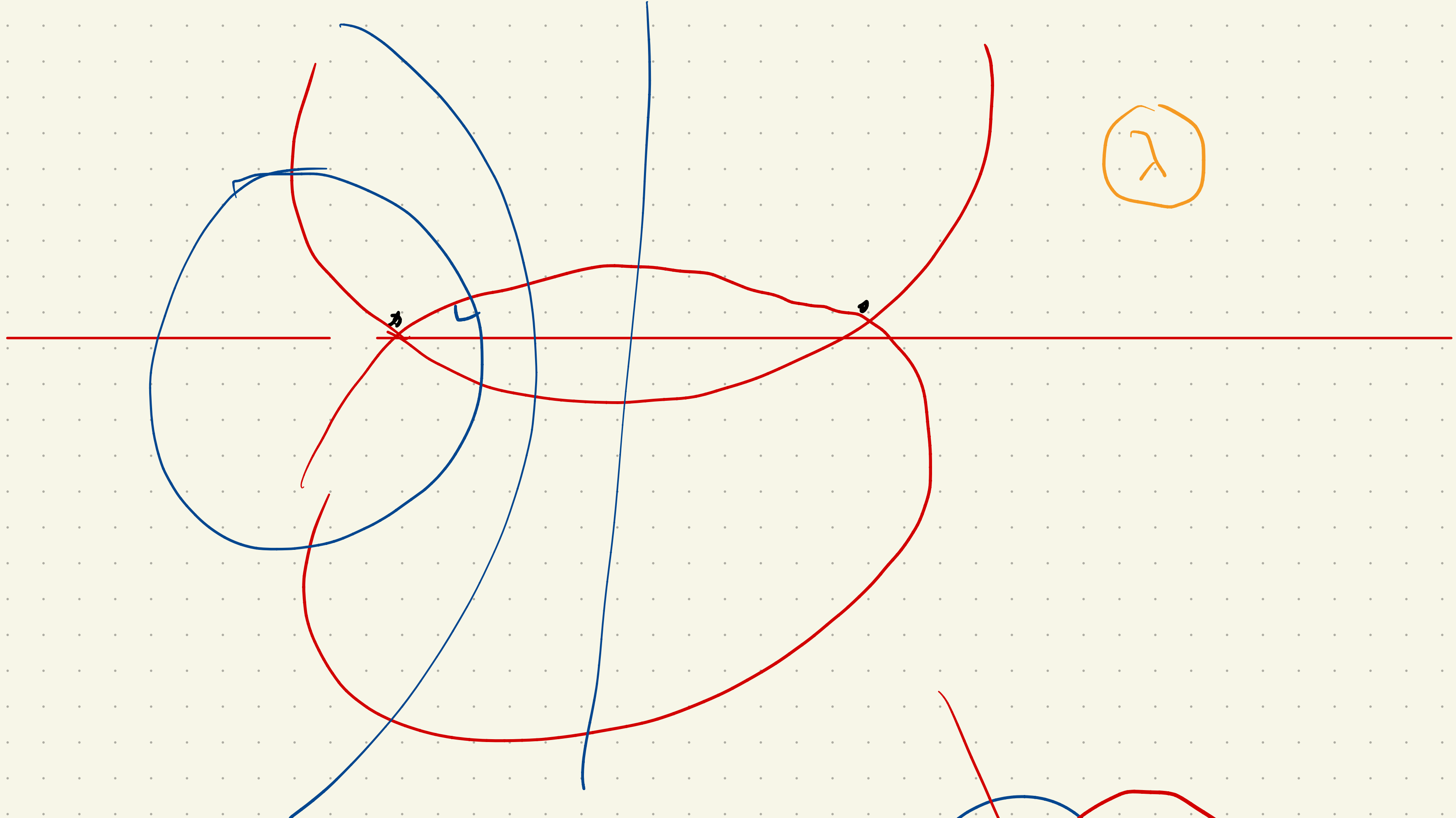
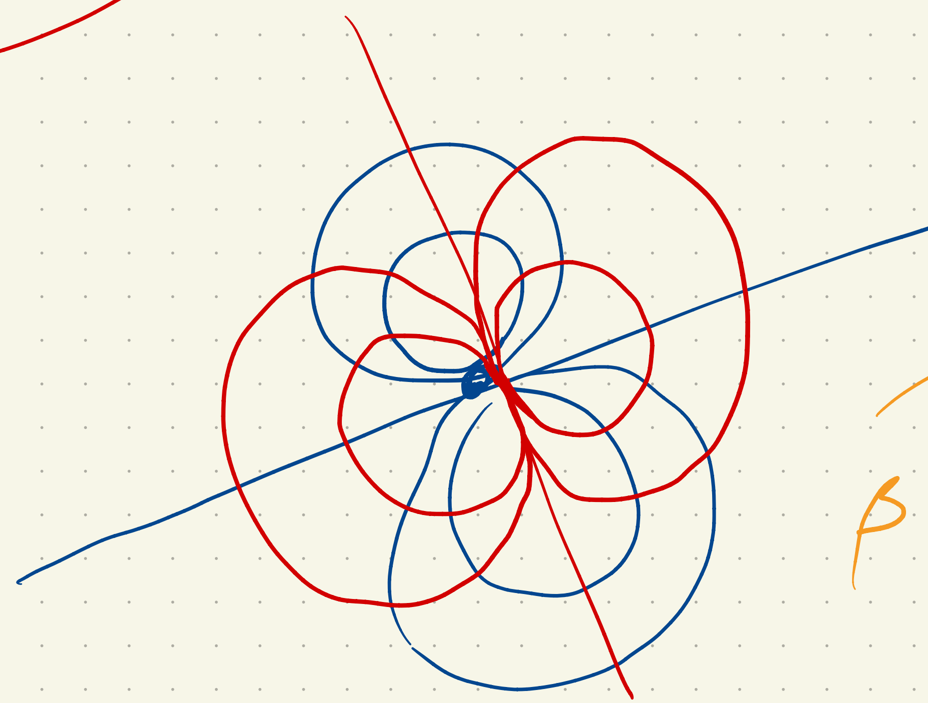
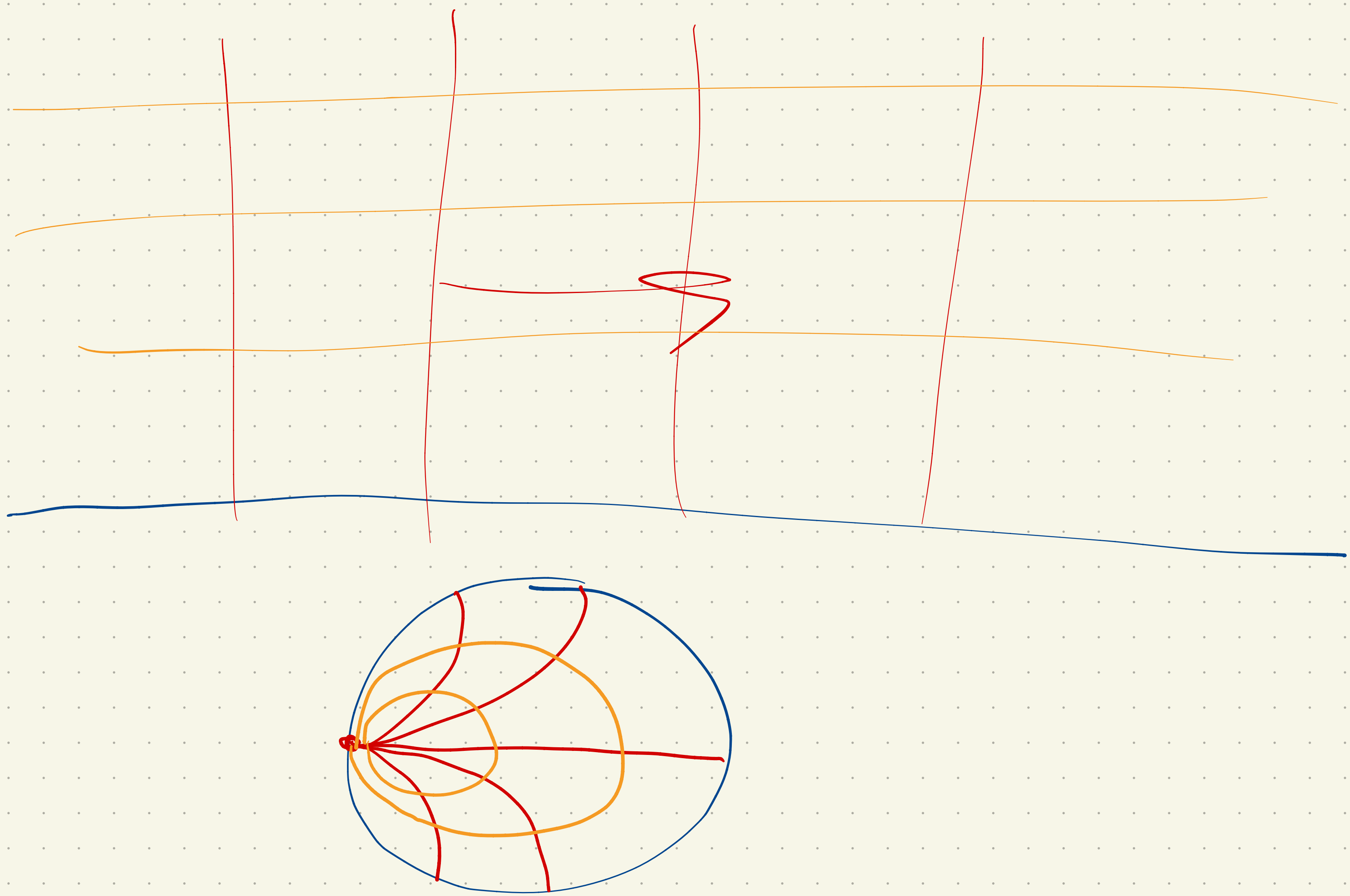


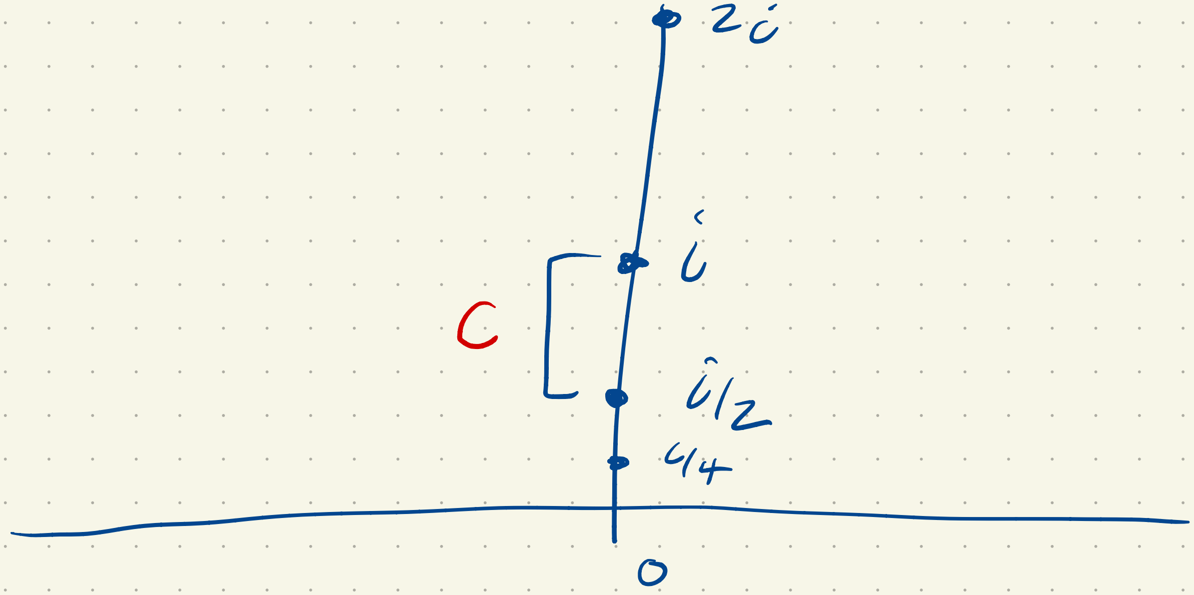
λ



β



Length



$z \mapsto \frac{1}{2} z$ is a hyperbolic transf.

$d(i, i/2) = C$

$d(2i, i) = C$ also if dist is preserved by hyp. transf.

If the vertical hyperbolic line is going to behave like a Euclidean line

$$d(2i,\ i/2) = 2C$$

$$d(2i,\ i/4) = 3C$$

$$d(2^j i,\ 2^k i) = |k-j|\, C$$

$$\ln\left(\frac{2^j i}{2^k i} \right) = \ln\left(2^{j-k}\right)$$

$$= (j-k)\ln(2)$$

$$\left| \ln\left(\frac{2^j i}{2^k i} \right) \right| = |j-k| \overbrace{|\ln(2)|}^{\ln(2)}$$

For points on the positive imaginary axis

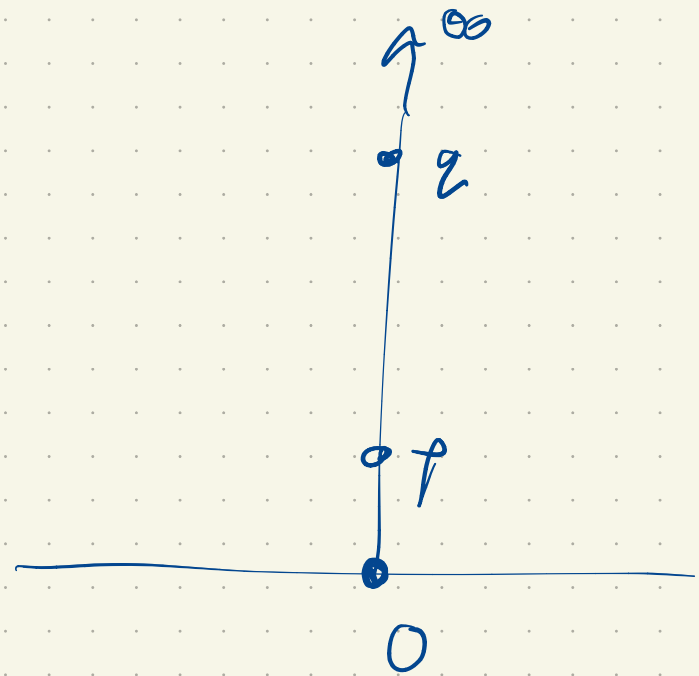
$$d(p,q) = \left| \ln\left(p/q \right) \right|$$

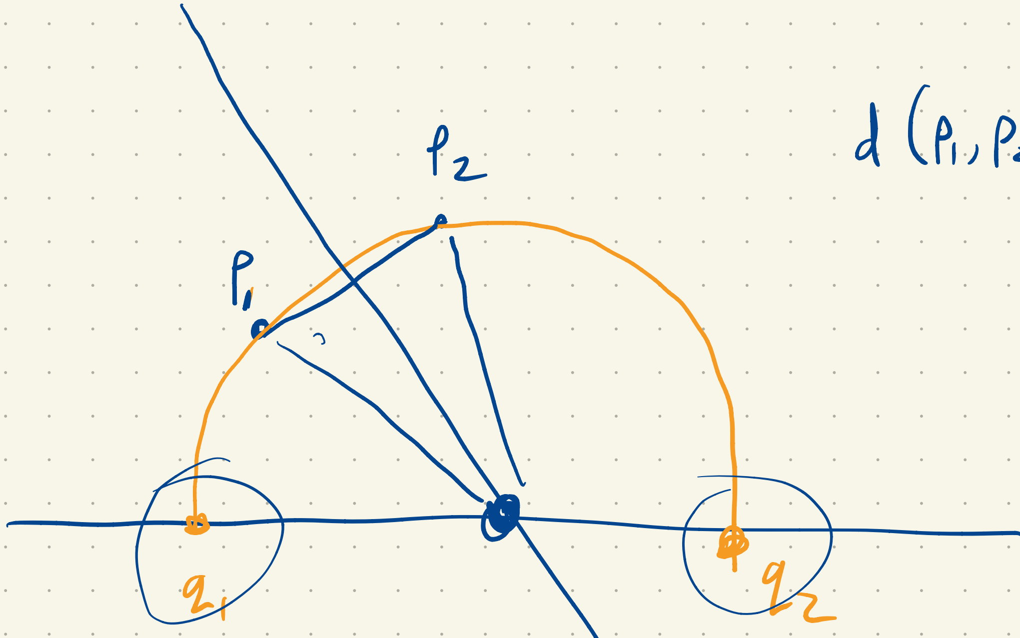
$$Tz = (z, \cdots)$$

$$(p, q, 0, \infty) = \frac{p-0}{p-\infty} \, \frac{q-\infty}{q-0} = p/q$$

$$\left| \ln\left((p, q, 0, \infty) \right) \right|$$

endpoints at ∞





$$d(p_1, p_2) = \left| \ln\left((p_1, p_2, \underbrace{q_1, q_2}) \right) \right|$$

endpoints at ∞ on the line joining p_1, p_2

$$\frac{p_1 - q_1}{p_1 - q_2} \cdot \frac{p_2 - q_2}{p_2 - q_1} = \left(\frac{p_1 - q_2}{p_1 - q_1} \cdot \frac{p_2 - q_1}{p_2 - q_2} \right)^{-1}$$

$$d(p_1, p_2) = d(Tp_1\ Tp_2) \qquad T : \text{hyp}$$

$$d(Tp_1, Tp_2) = \left| \ln\left((Tp_1, Tp_2, Tq_1, Tq_2) \right) \right.$$

$$= \left| \ln\left((p_1, p_2, q_1, q_2) \right) \right|$$

$$= d(p_1, p_2)$$