Hyperbolic transformations.

Möbius transformations $\longrightarrow$ at most two (unless the id)
one is possible, but not none.

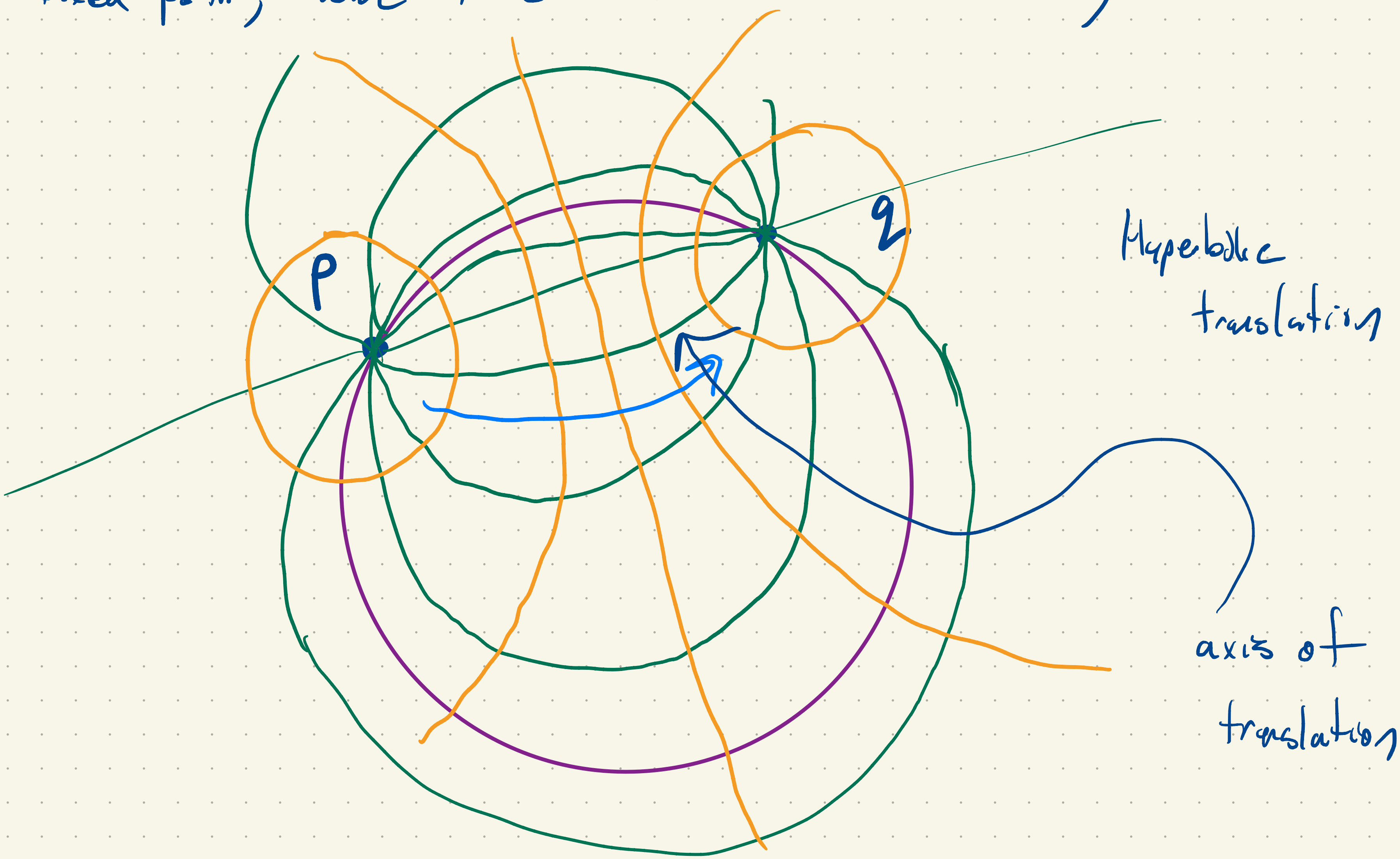
Suppose T is a hyperbolic transformation
and $p \notin S^1$ is a fixed point.

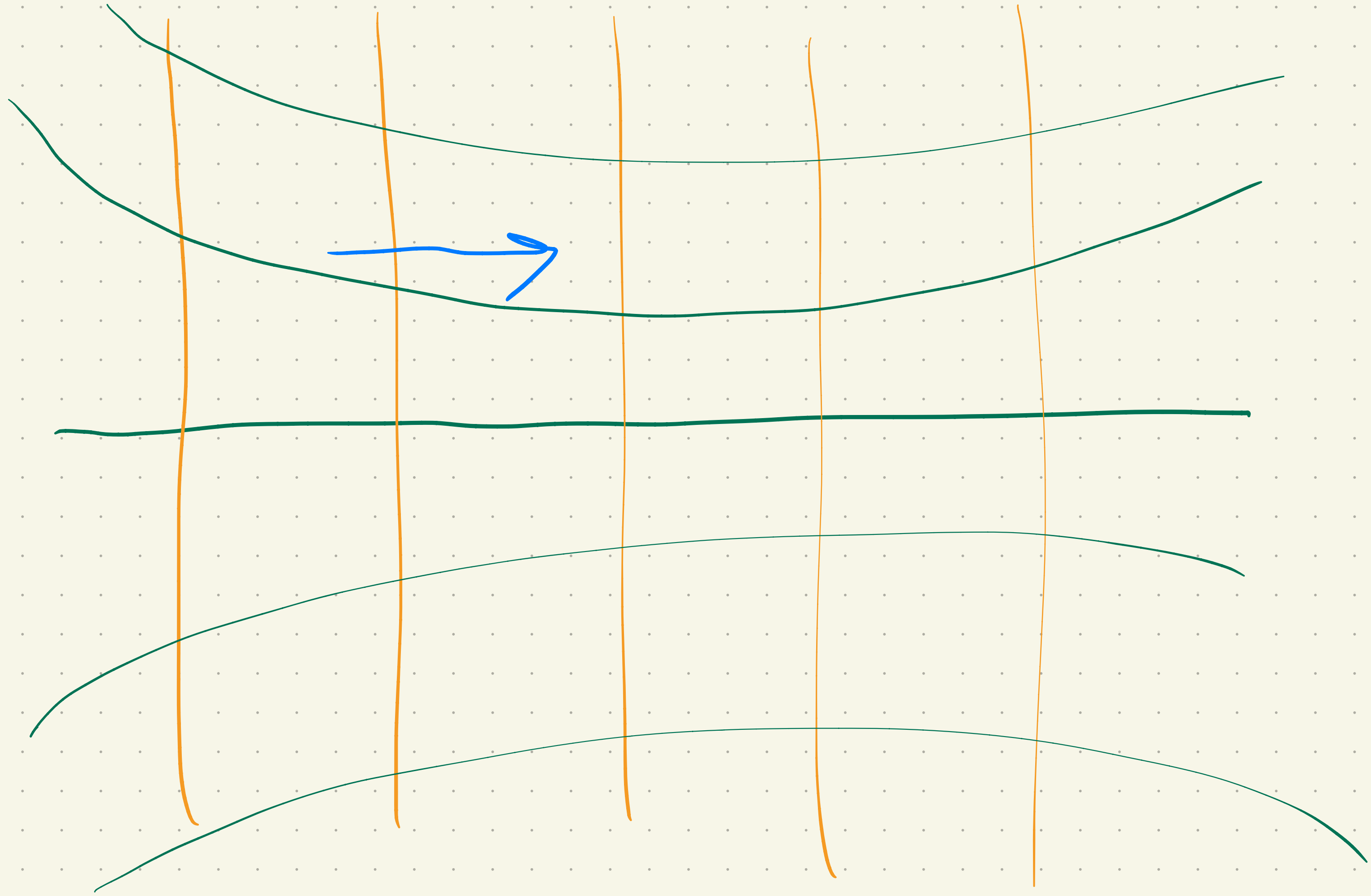
$$T(p) = p$$

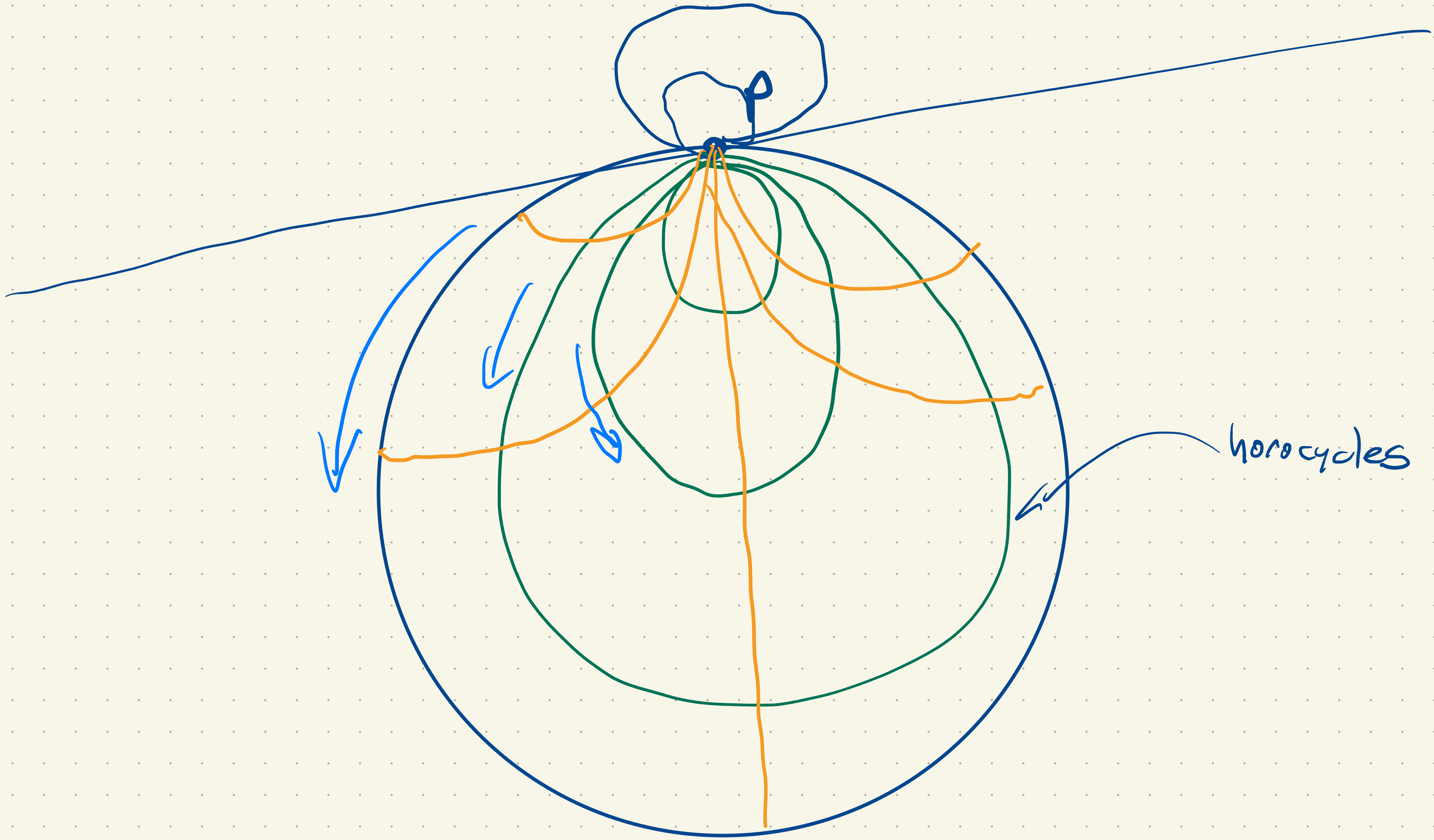
$$T(p^*) = \left(T(p)\right)^* = p^*$$

$$T(z) = \lambda z \qquad \lambda = e^{i\theta}$$

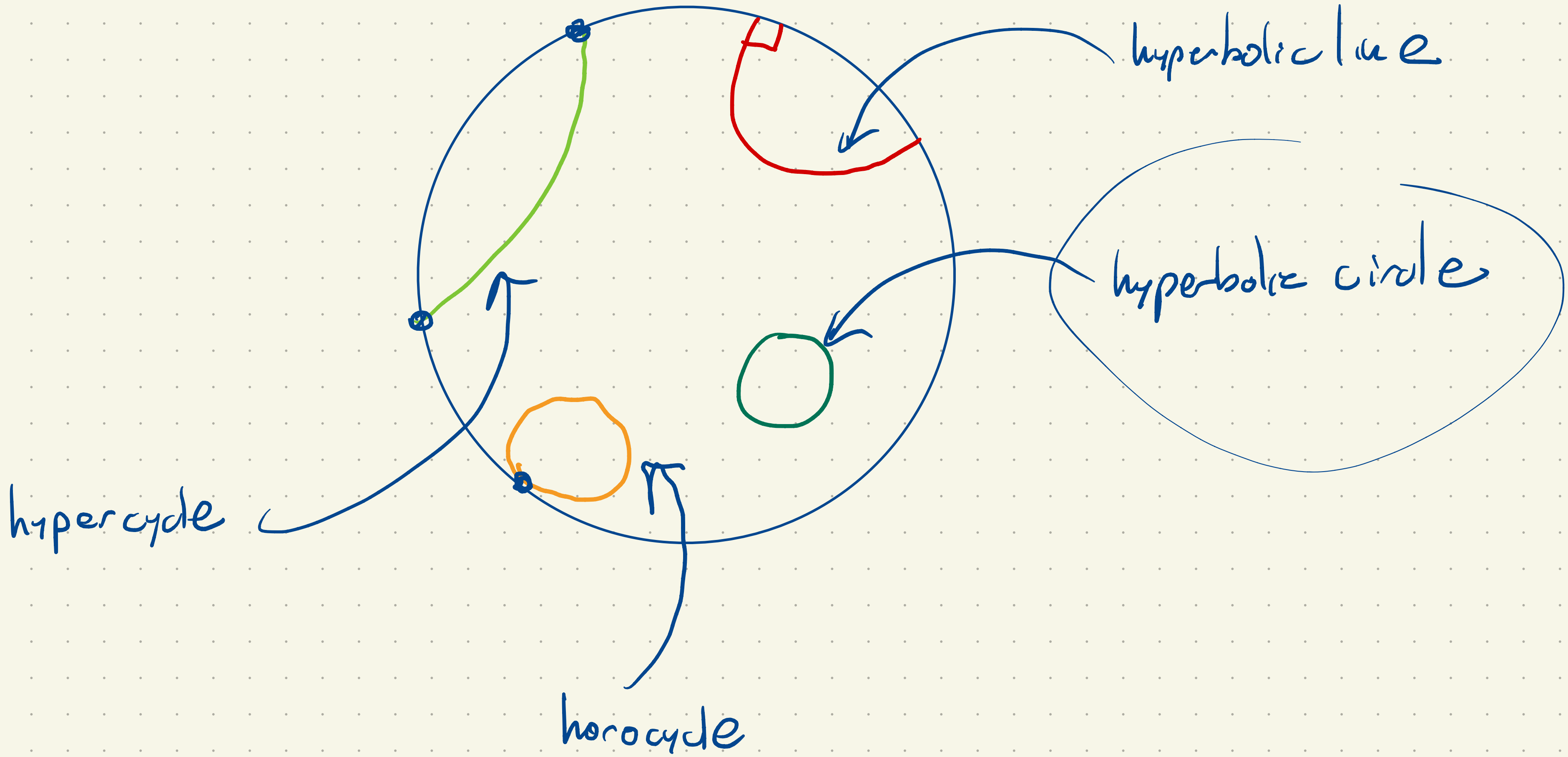
2 fixed points, none in D (so both on S^1)



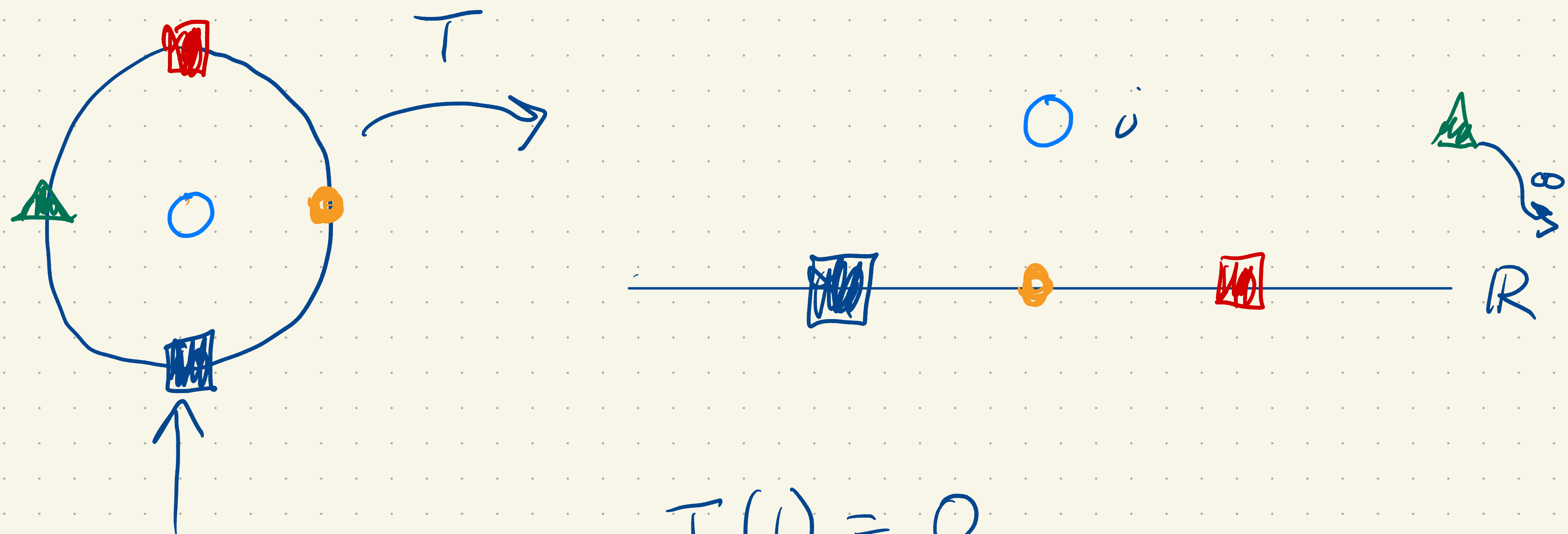




P
horocycles


hyperbolic line
hyperbolic circle
hypercycle
horocycle

Half plane model



Poincaré Disc Model

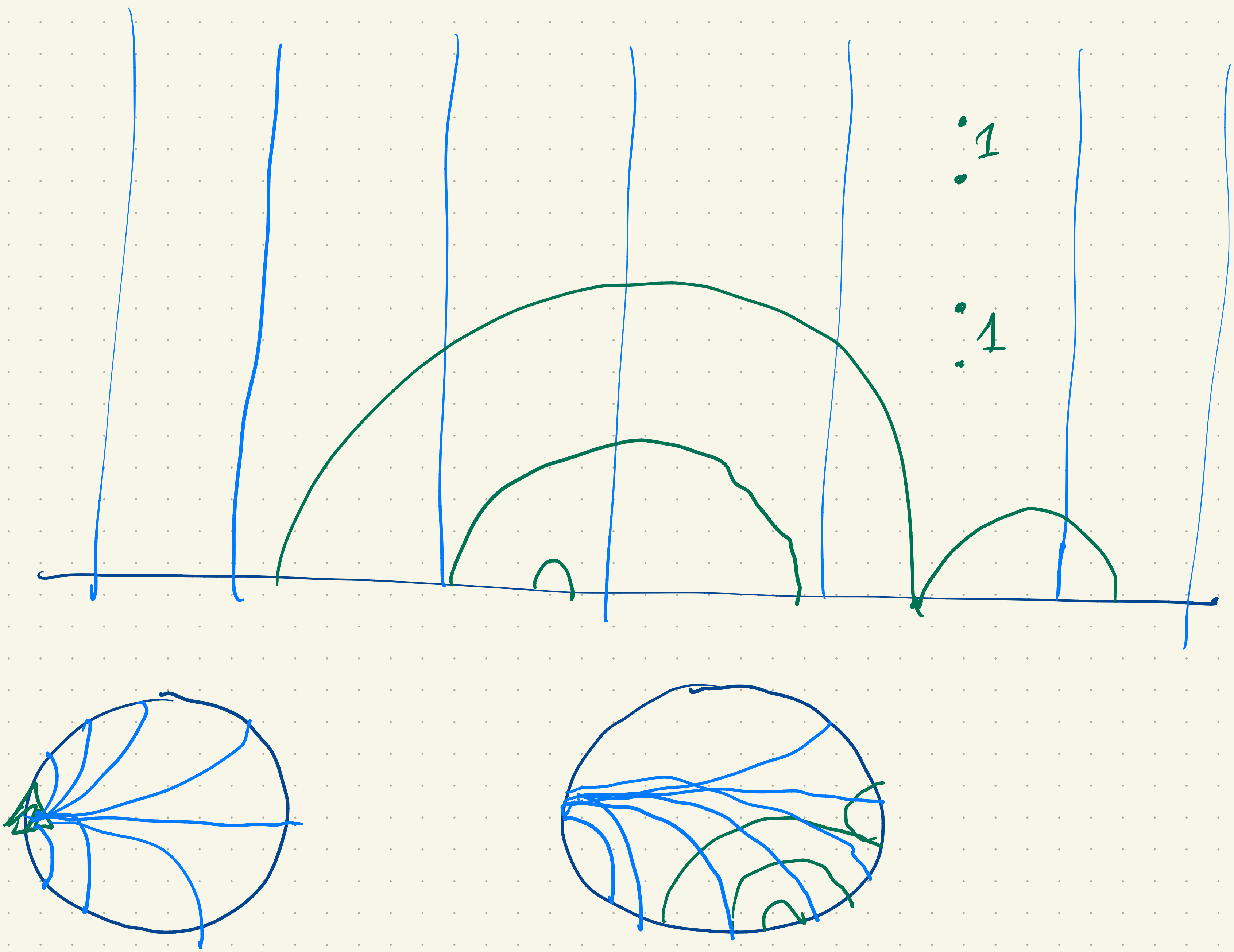
$$T(1) = 0$$

$$T(i) = 1 \qquad (z, i, 1, \infty)$$

$$T(-1) = \infty$$

$$T(z) = \frac{z-1}{z+1} \, \frac{i+1}{i-1} = i\left[\frac{1-z}{1+z}\right]$$

$$T(-i) = i\left(\frac{1-(-i)}{1+(-i)}\right) = \frac{i + i \cdot i}{1 - i} = \frac{i-1}{1-i} = -1$$



$\mathbb{C}^+ = \mathbb{C} \cup \{\infty\}$

Hyperbolic transformations:

Möbius transformations that take upper half plane to upper half plane and

by continuity take $\mathbb{R}^+$ to $\mathbb{R}^+$.

$$T(z) = \frac{az + b}{cz + d}$$

$ad - bc \neq 0$

If $a, b, c, d \in \mathbb{R}$ $(ad - bc \neq 0)$

then $\mathbb{R}^+ \to \mathbb{R}^+$