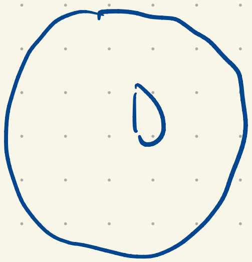


$p \in D$

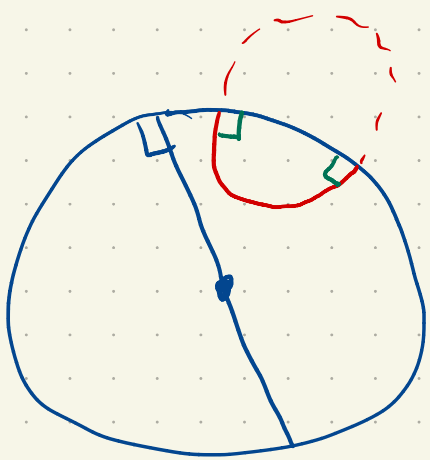
$\lambda \in S^1 \rightarrow e^{i\theta}$ for some $\theta \in \mathbb{R}$

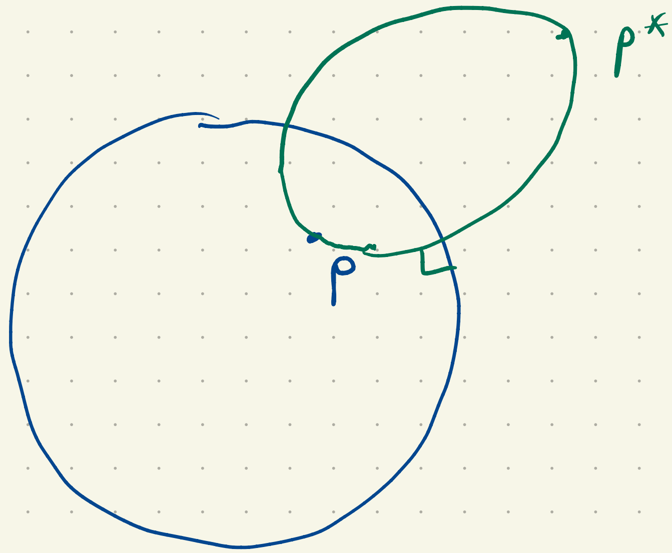
$$Tz = \lambda \left(\frac{z - p}{1 - z\bar{p}} \right)$$

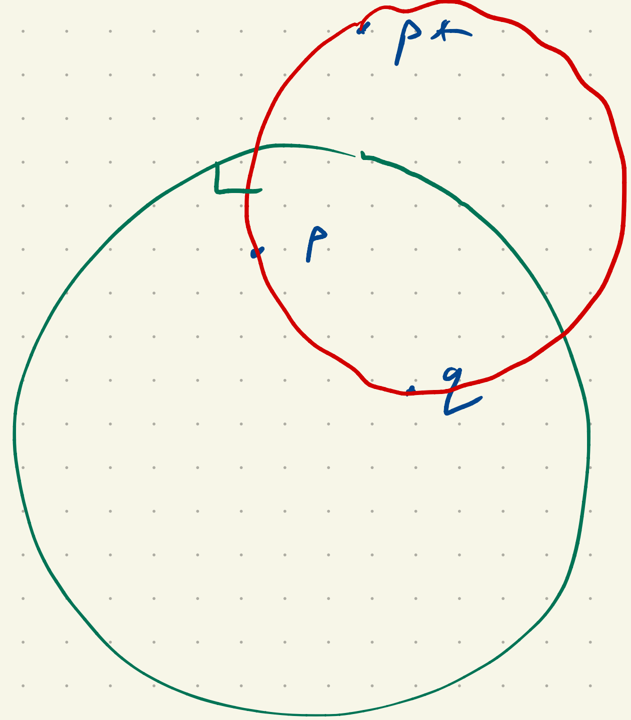
$\uparrow$

hyperbolic transformations

hyperbolic line:

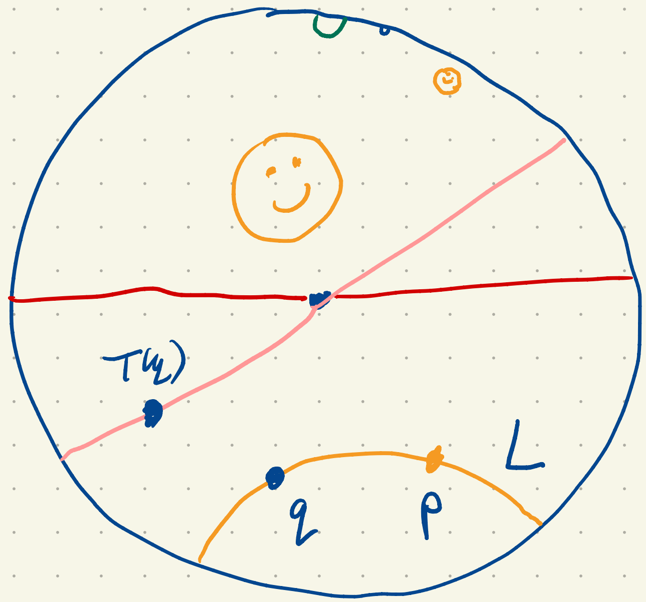






Given distinct points $p, q \in \mathbb{D}$ there is a unique hyperbolic line incident to both.

All hyperbolic lines are congruent



$$T(p) = 0$$
$$T(q) =$$

$$\frac{z-p}{1-z\bar{p}}$$

$$\lambda \frac{z-p}{1-z\bar{p}} \rightarrow \lambda z \qquad p = 0$$

$$S(z) = \frac{\overline{T(q)}}{|T(q)|} z$$

$$S(T(q)) = \frac{\overline{T(q)}}{|T(q)|} T(q) = \frac{|T(q)|^2}{|T(q)|} = |T(q)| \in \mathbb{R}$$

So T is a hyperbolic transformation

taking p to 0

q to somewhere on $\mathbb{R}$.

$T(L)$ is a hyperbolic line passing through 0 and some non zero real number.

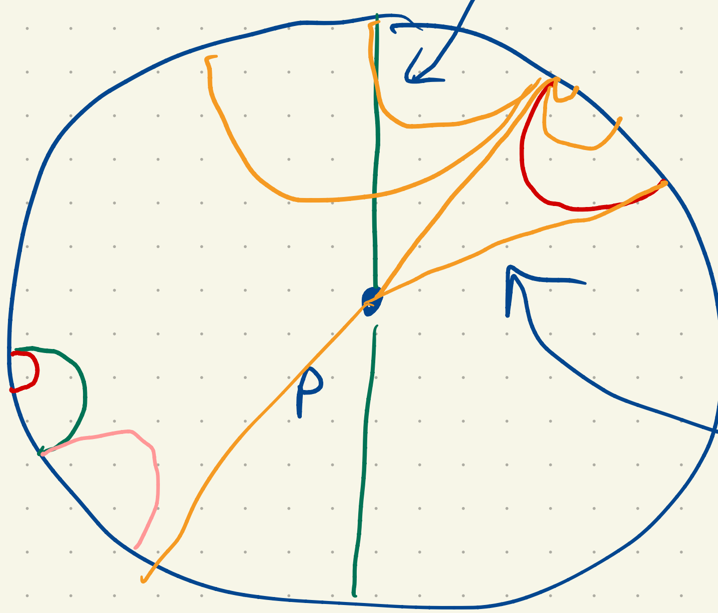
So it's $\mathbb{R}^1$.

Points on S^1 are called ideal points.

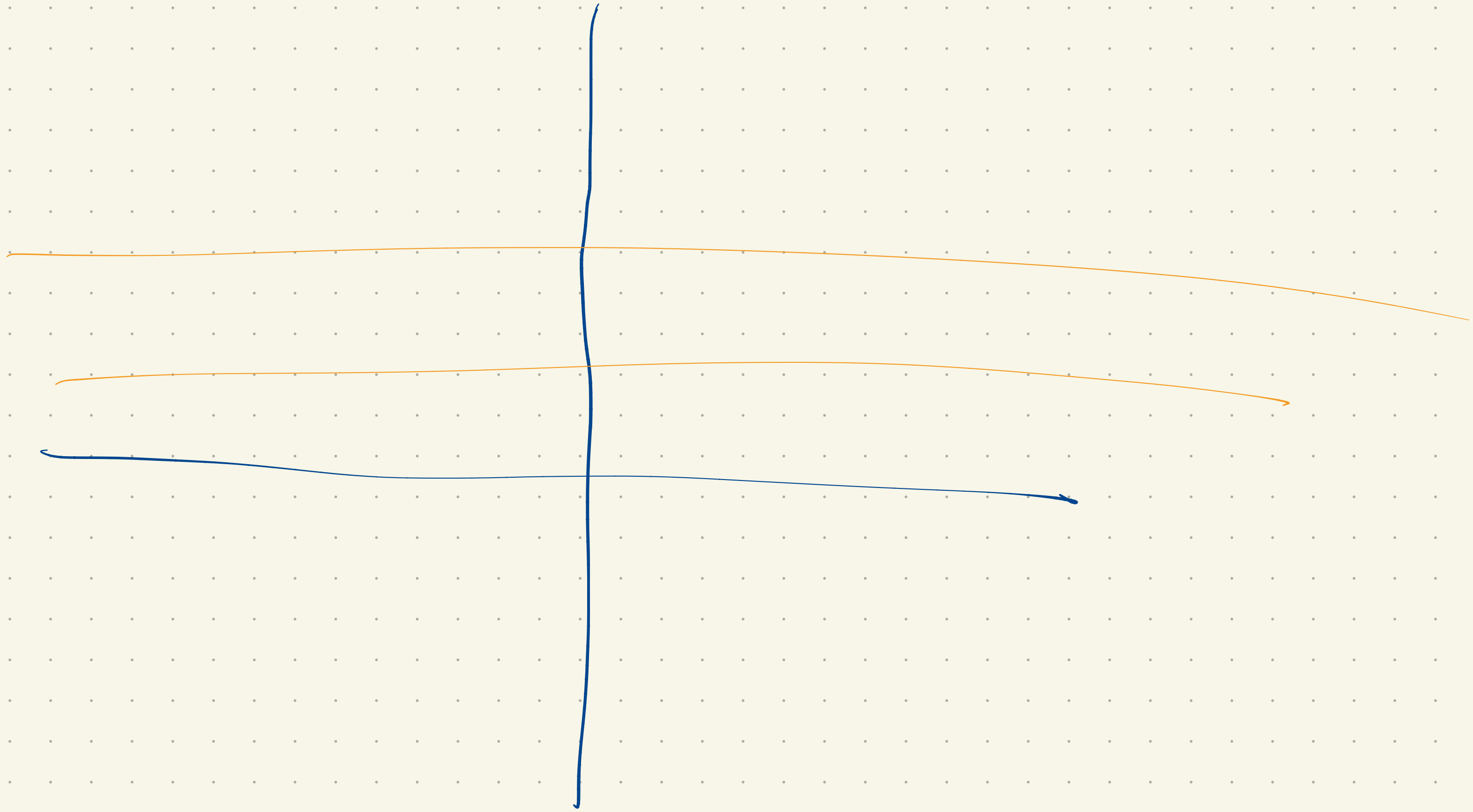
Think of them as "points at ∞"

Parallelism Given a line and a point not on the line how many parallel lines are there through the point?

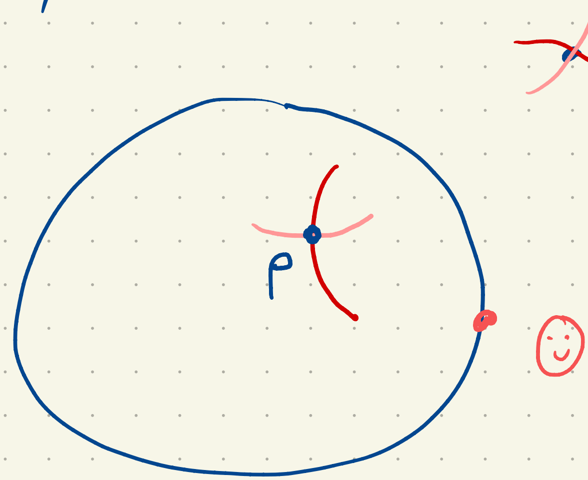
hyperparallel



parallel (meet at an ideal point)



How many times can two hyperbolic lines intersect on S^1?

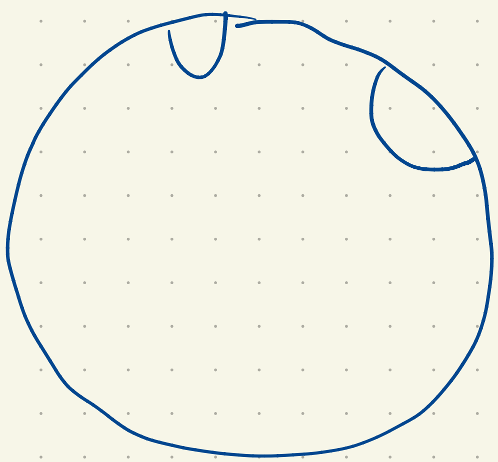


If two lines intersect at $p \in D$ they also intersect at p^*.

If they intersect at ☺ $\in S^1$ then they are the same Möbius line.

Upshot: Two distinct lines that intersect in $\mathbb{D}$ cannot intersect on S^1!

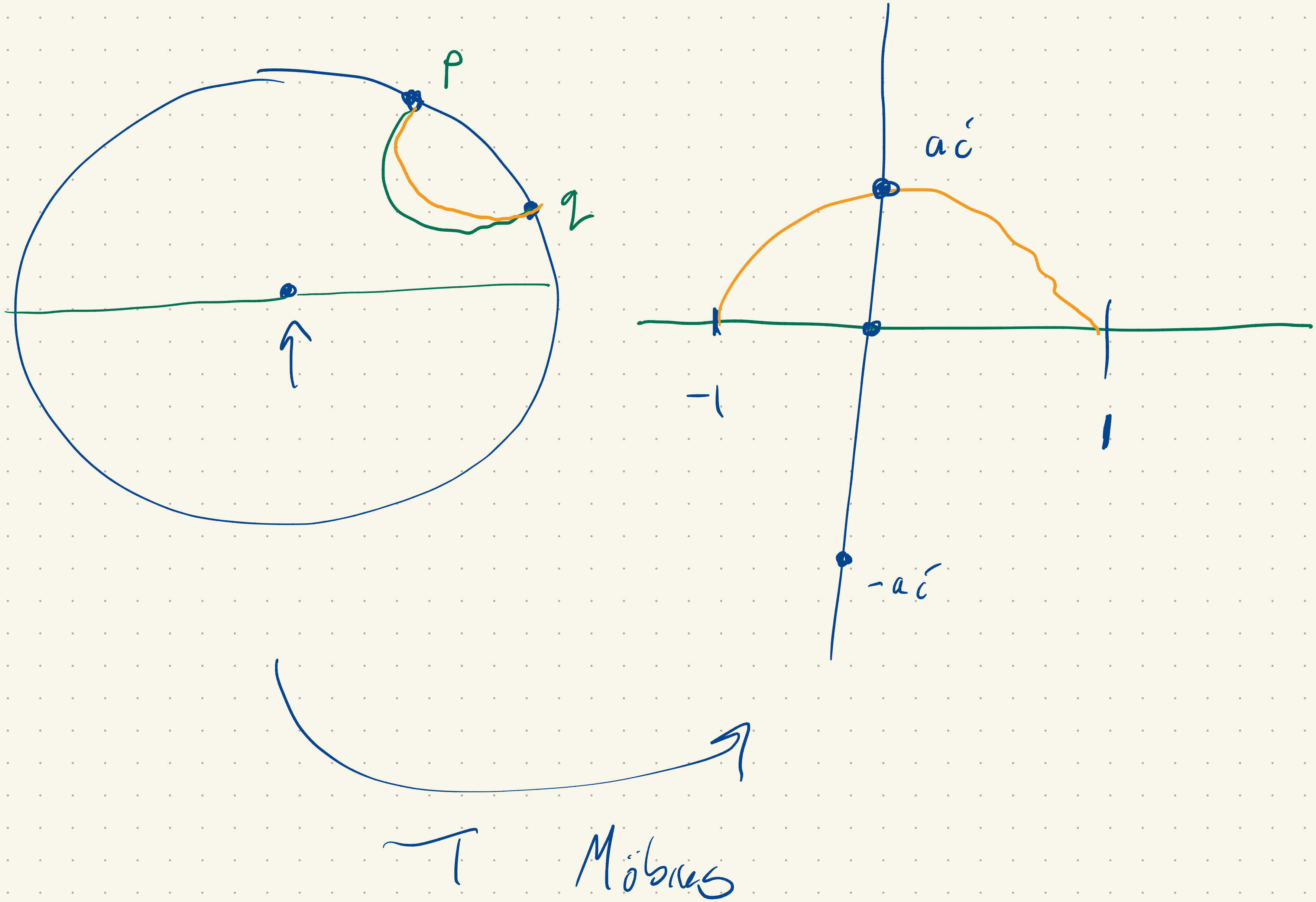
Contrapositive: If two hyperbolic lines intersect at a point on S^1 they do not intersect in $\mathbb{D}$.



Two lines can intersect no more than twice on the boundary.

Is just once possible? Sure.

Is twice possible



$(1, -1, ai, -ai)$ is Real

$\Rightarrow a = 0, 1, -1$