$$
\begin{array}{lll}
p \to 0 & p \to \infty & (p, q, \lambda) \\
q \to \infty & q \to 0 & (q, p, \lambda^{-1}) \\
\lambda & 1/\lambda &
\end{array}
$$

One fixed point p

$$S(z) = \frac{1}{z-p} \qquad \left(\lambda_0 \frac{1}{z-p} + b_0 \right)$$

$$
\begin{tikzcd}
\mathbb{C}^+ \arrow[r, "R"] \arrow[d, "S^{-1}"'] & \mathbb{C}^+ \\
\mathbb{C}^+ \arrow[r, "T"] & \mathbb{C}^+ \arrow[u, "S"']
\end{tikzcd}
$$

$$S(z) = w \qquad \frac{1}{z-p} = w$$

$$z = S^{-1}(w)$$

R is a Möbius transformation with 1 fixed point, ∞

$$R(z) = \frac{az+b}{cz+d}$$

$$R(\infty) = \frac{a}{c}$$

$c=0 \Rightarrow R(\infty)=\infty$

$ad-bc \neq 0$

$\uparrow$

$$R(z) = \frac{az+b}{d} = \frac{a}{d}z + \frac{b}{d}$$

$ad \neq 0$

$$R(z) = az+b$$

$az+b=z$

$az-z=-b$

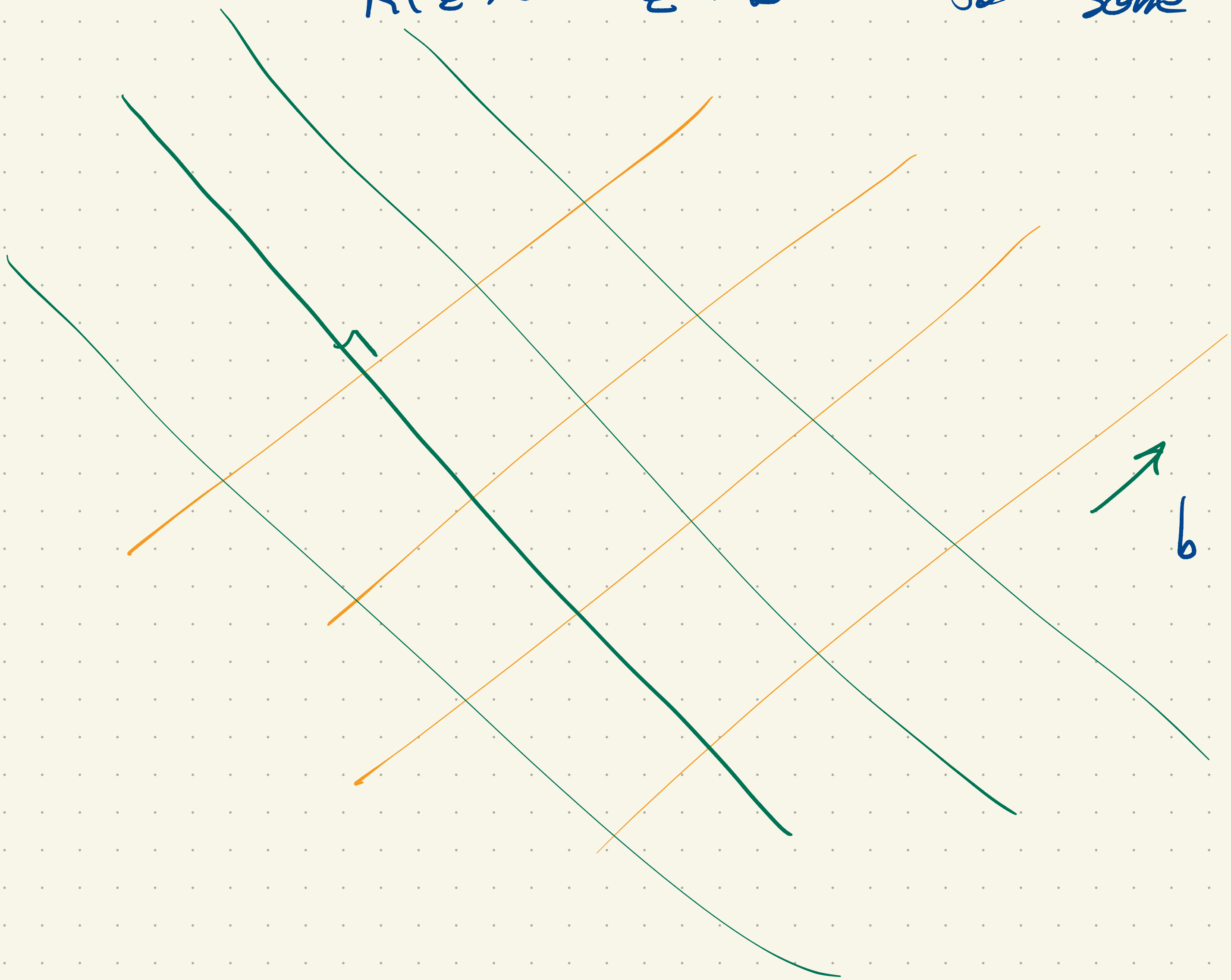
$$z = -\frac{b}{a-1}$$

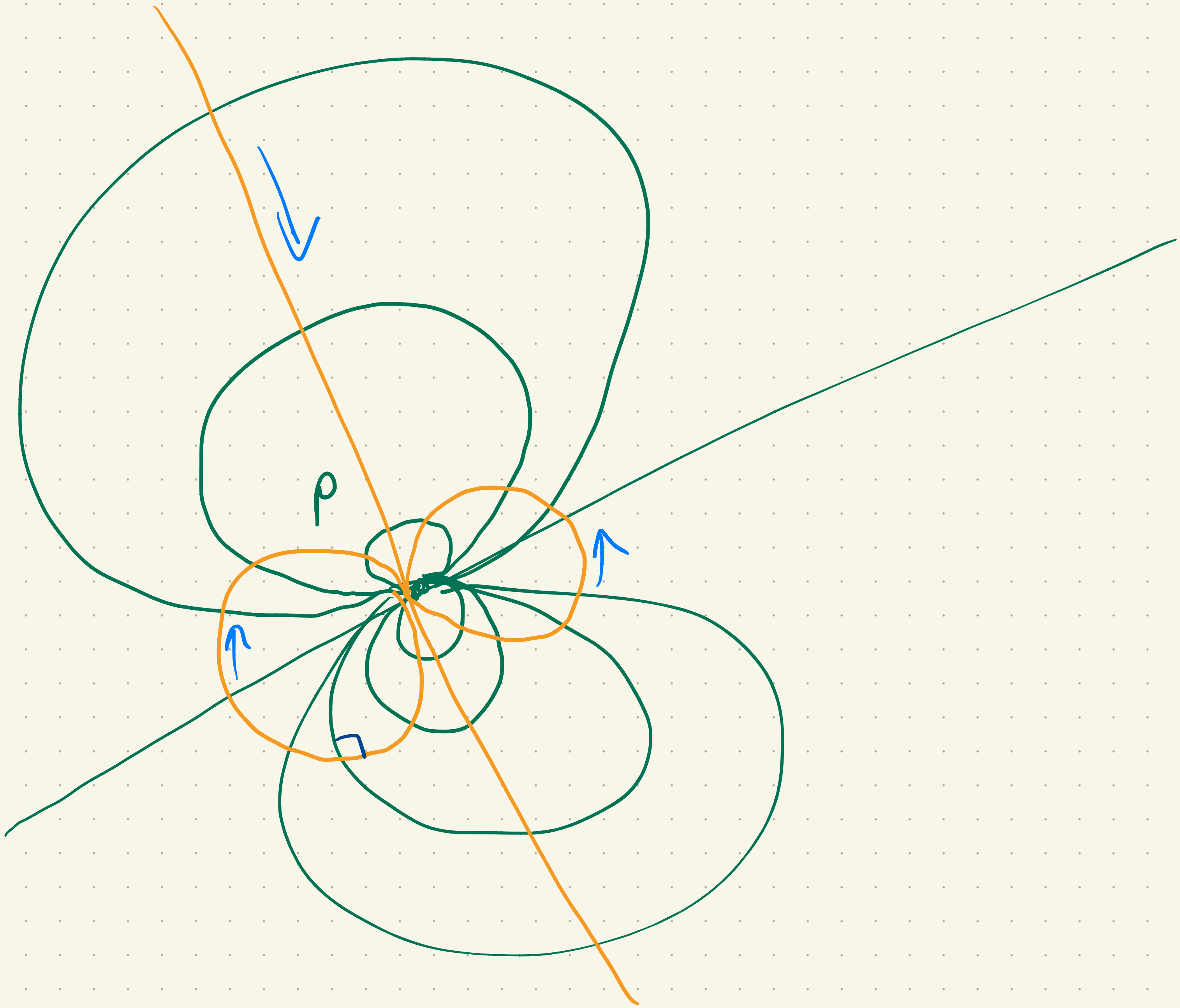
$z+b=z$

$b=0$

If R has exactly one fixed point, ∞, then

$$R(z) = z + b \quad \text{for some } b \in \mathbb{C}, \ (b \neq 0)$$




ρ

$R = S \circ T \circ S^{-1}$

$$S(z) = \frac{1}{z-p}$$

$$\underbrace{R \circ S}_{} = \underbrace{S \circ T}_{}$$

$$R(z) = z + b$$

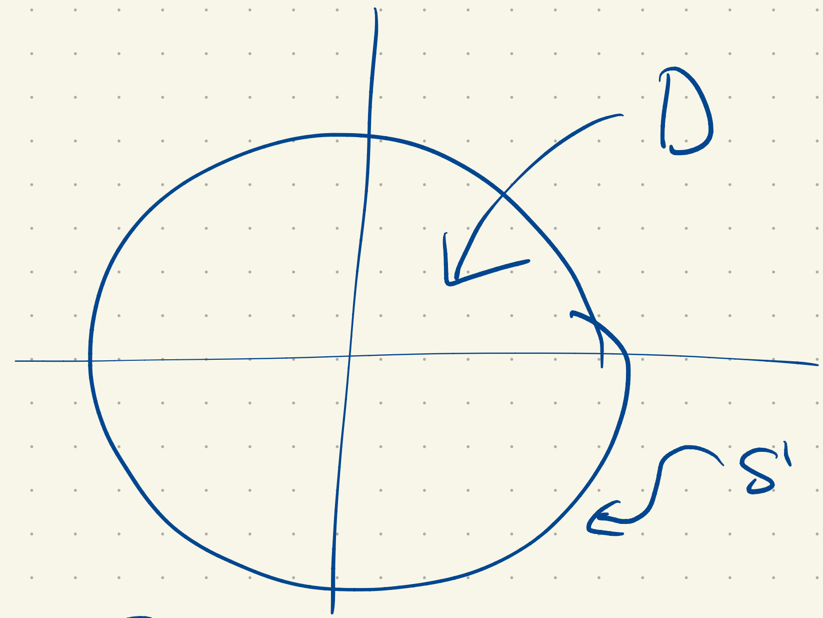
$$\boxed{\frac{1}{z-p} + b = \frac{1}{Tz - p}}$$

Normal form

(1 fixed point)

Challenge: To what extent is b determined by T?

Hyperbolic Geometry



$$D = \{ z \in \mathbb{C} : |z| < 1 \}$$

$$S^1 = \{ z \in \mathbb{C} : |z| = 1 \}$$

$$\mathcal{G} = \{ \text{Möbius transformations } T : \underbrace{T(D) = D} \}$$

$T(D) \subseteq D$

$T(D) = D$

If $z \in D$ $\quad Tz \in D$

If $w \in D$ there is $z \in D$ $\quad Tz = w$.

Is this a trans. group? Yes!

a) id ✓

b) closed under composition ✓

c) closed under inverses ✓

Who are these?

$T(D) = D$ implies there is some $p \in D$ with $Tp = 0$