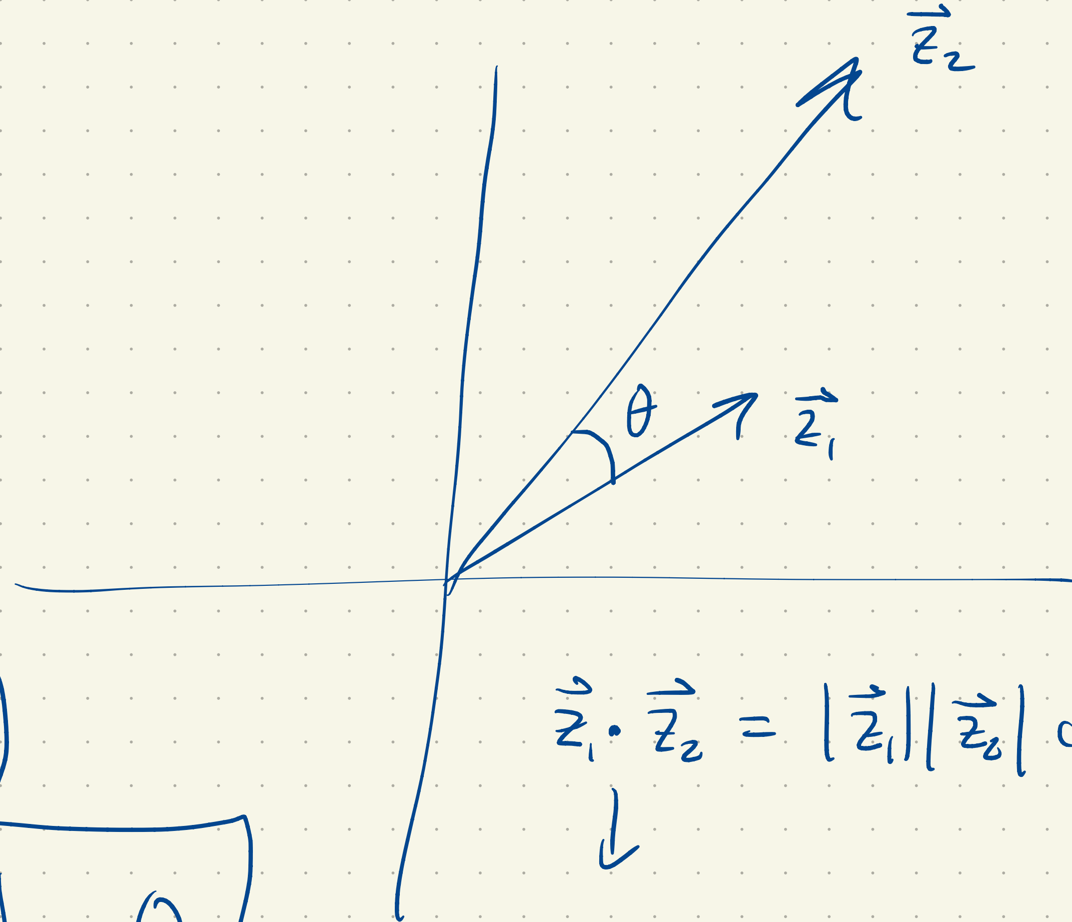
$$z_1 = x_1 + iy_1$$

$$z_2 = x_2 + iy_2$$

$$z_1\overline{z_2} = (x_1x_2 + y_1y_2) + i(-x_1y_2 + y_1x_2)$$

$$\boxed{\mathrm{Re}(z_1\overline{z_2}) = |z_1||z_2|\cos\theta}$$

$$\cos\theta = \frac{\mathrm{Re}(z_1\overline{z_2})}{|z_1||z_2|}$$



$$\vec{z}_1 \cdot \vec{z}_2 = |\vec{z}_1||\vec{z}_2|\cos\theta$$

$$\downarrow$$

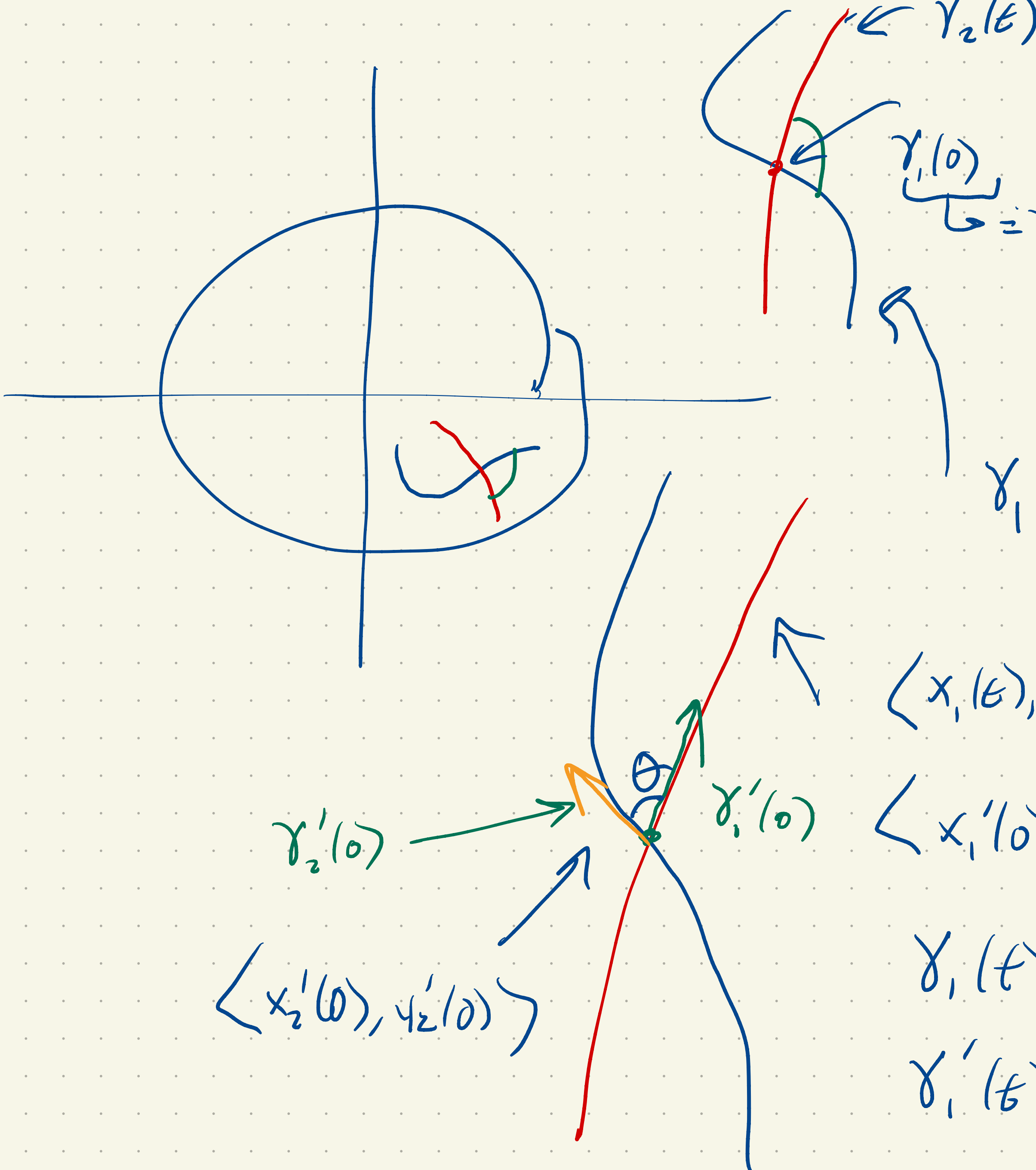
$$x_1x_2 + y_1y_2$$

$$0 \le \theta \le \pi$$

$$\frac{\pi}{2}$$

Claim: I preserves angles

"conformal"



$$z \to \frac{1}{z}$$

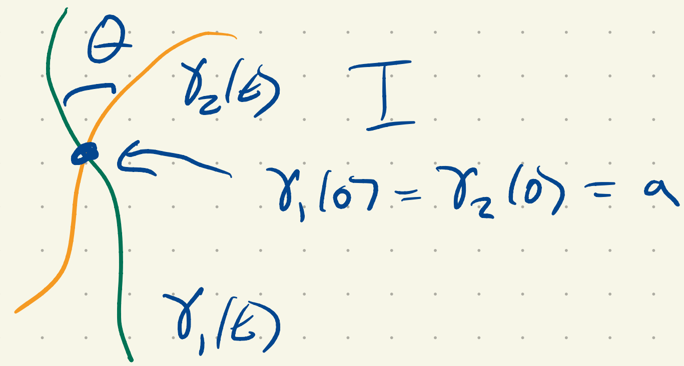
$$\langle x_1(t), y_1(t) \rangle$$

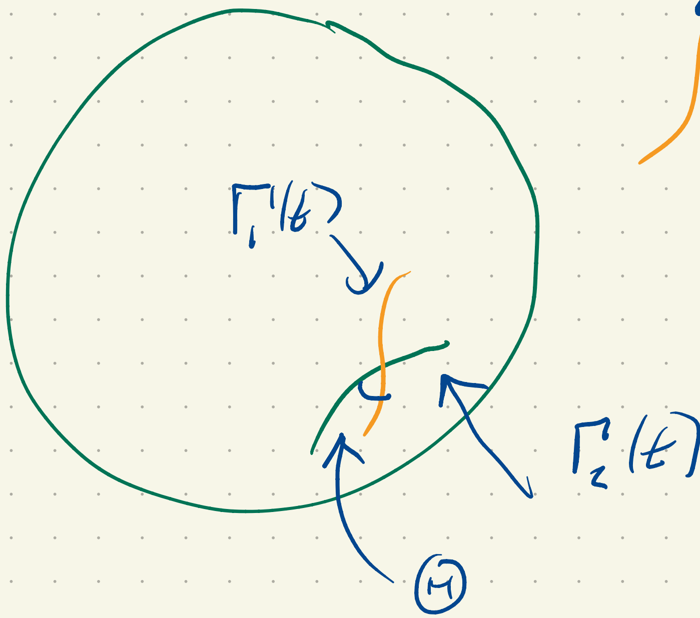
$$\langle x_1'(0), y_1'(0) \rangle$$

$$\gamma_1(t) = x_1(t) + i\, y_1(t)$$

$$\gamma_1'(t) = x_1'(t) + i\, y_1'(t)$$

$$\cos\theta = \frac{\operatorname{Re}\left(\gamma_1'(0)\,\overline{\gamma_2'(0)}\right)}{|\gamma_1'(0)|\,|\gamma_2'(0)|}$$





$$\Gamma_1(t) = \frac{1}{\gamma_1(t)}$$

$$\Gamma_2(t) = \frac{1}{\gamma_2(t)}$$

$$\frac{d}{dt}\Gamma_1(t) = \frac{d}{dt}\frac{1}{\gamma_1(t)} = -\frac{1}{\gamma_1^2(t)}\gamma_1'(t) \qquad \frac{d}{dx}\frac{1}{x} = -\frac{1}{x^2}$$

$$\Gamma_1'(0) = -\frac{1}{\gamma_1^2(0)}\gamma_1'(0) = -\frac{1}{a^2}\gamma_1'(0)$$

$$\Gamma_2'(0) = \frac{-1}{\gamma_2^2(0)}\gamma_2'(0) = \frac{-1}{a^2}\gamma_2'(0)$$

$$\cos\Theta = \frac{\mathrm{Re}\left(\Gamma_1'(0)\overline{\Gamma_2'(0)}\right)}{|\Gamma_1'(0)|\,|\Gamma_2'(0)|}$$

$$|\Gamma_1'(0)| = \left|\frac{-1}{a^2}\gamma_1'(0)\right|$$

$$= \frac{1}{|a|^2}|\gamma_1'(0)|$$

$$|\Gamma_2'(0)| = \frac{1}{|a|^2}|\gamma_2'(0)|$$

$$\underbrace{\Gamma_1'(0)\overline{\Gamma_2'}(0)} = \frac{-1}{a^2}\gamma_1'(0)\left(\frac{-1}{(\bar{a})^2}\overline{\gamma_2'}(0)\right)$$

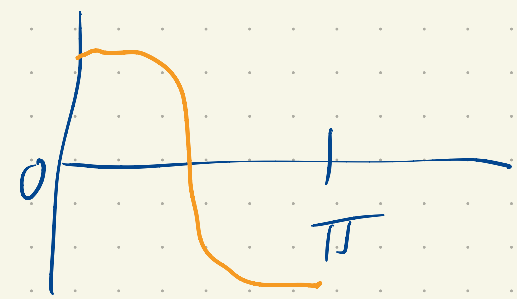
$$= \frac{1}{|a|^4}\gamma_1'(0)\overline{\gamma_2'(0)}$$

$$\mathrm{Re}(\text{——}) = \frac{1}{|a|^4}\mathrm{Re}\left(\gamma_1'(0)\overline{\gamma_2'(0)}\right)$$

$$\cos \Theta = \frac{|a|^4}{|\gamma_1'(0)|\,|\gamma_2'(0)|} \cdot \frac{1}{|a|^4} \operatorname{Re}\left(\gamma_1'(0)\,\overline{\gamma_2'(0)}\right)$$

$$\frac{5 - 2z + z^3}{6 - z}$$

$$= \frac{\operatorname{Re}\left(\gamma_1'(0)\,\overline{\gamma_2'(0)}\right)}{|\gamma_1'(0)|\,|\gamma_2'(0)|} = \cos \Theta$$

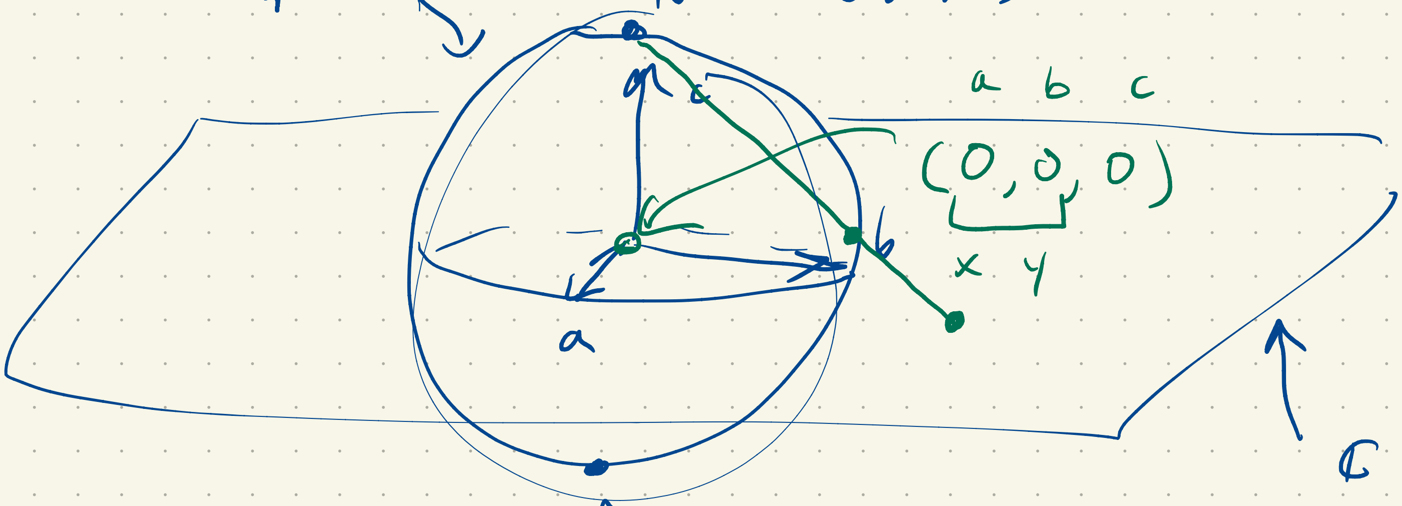


Stereographic Projection

$(x,y) \longleftrightarrow x+iy$

Riemann sphere

$N = (0,0,1)$



$S = (0,0,-1)$

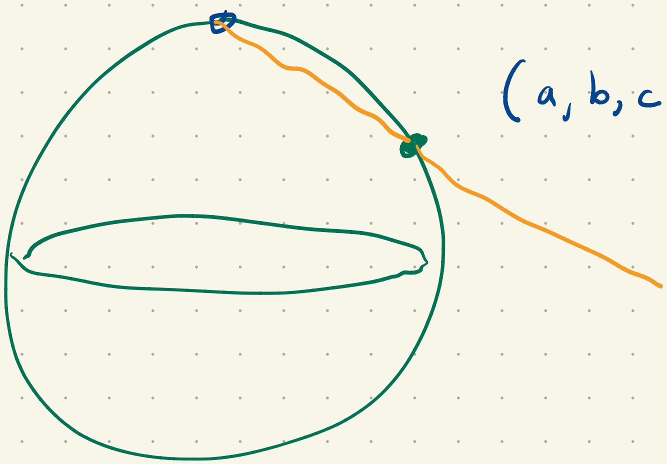
$$\mathbb{C} \cup \{\infty\} = \overline{\mathbb{C}}$$

$$\frac{1}{\infty} := 0$$

$$\frac{1}{0} := \infty$$

$-\infty \longleftarrow 0 \longrightarrow +\infty$

$N = (0,0,1)$



(a, b, c)

$$a^2 + b^2 + c^2 = 1$$

$$(0,0,1) + \lambda(a, b, c-1)$$

$\downarrow$

$\lambda \in \mathbb{R}$

$$\left(\lambda a, \lambda b, 1+\lambda(c-1)\right) = \left(\overset{a}{\cancel{x}}, \overset{b}{\cancel{x}}, 0\right)$$

$$x \longmapsto \sin(x^2)$$

$$1+\lambda(c-1) = 0$$

$$N = (0,0,1)$$

$$\lambda(c-1) = -1$$

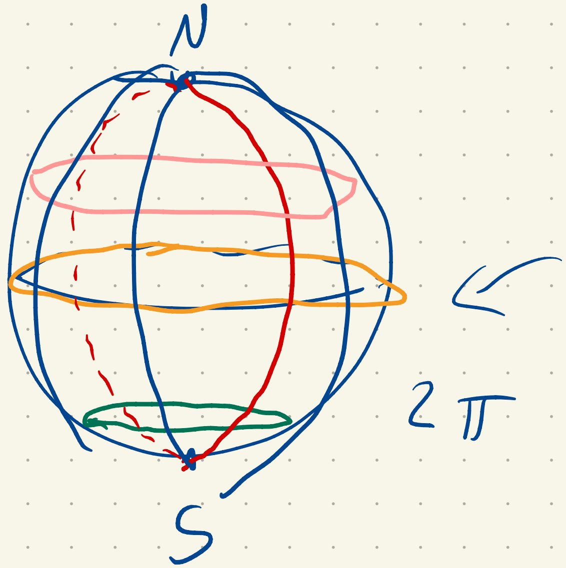
$$\lambda = \frac{-1}{c-1} = \frac{1}{1-c}$$

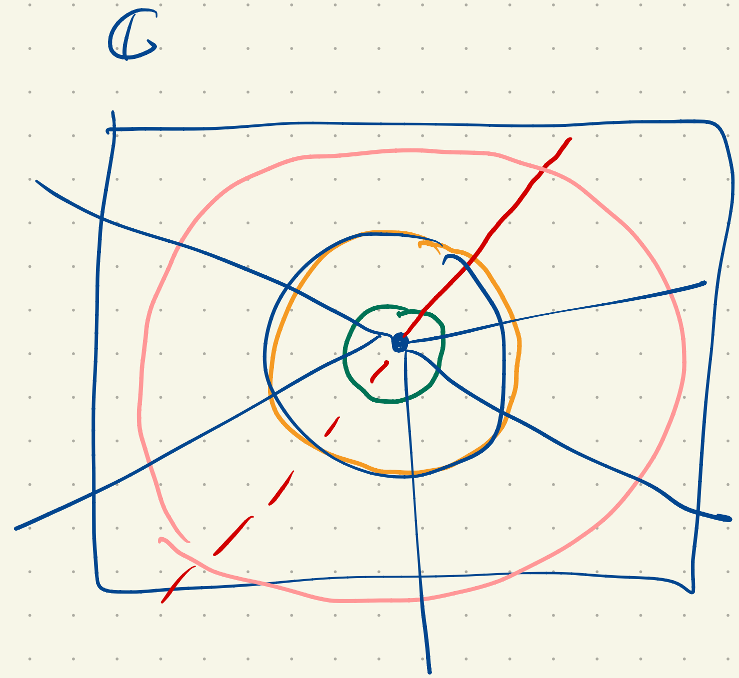
$$\left(\underbrace{\frac{a}{1-c}}_{x}, \underbrace{\frac{b}{1-c}}_{y}, 0\right)$$

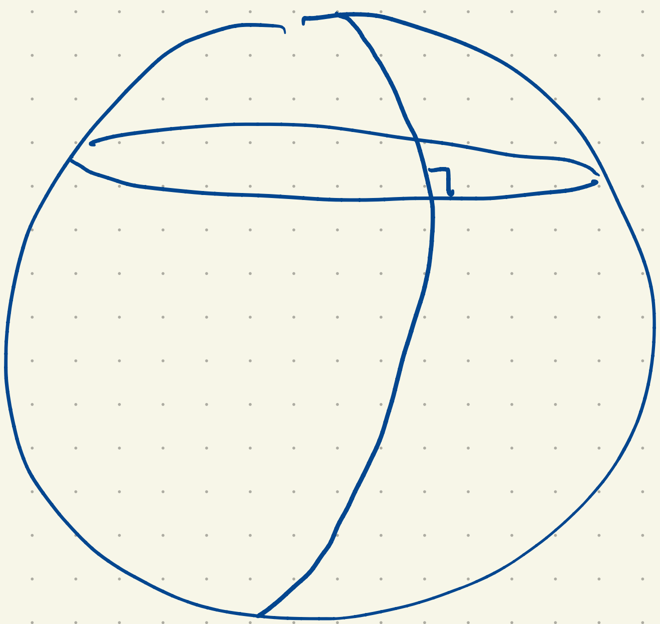
$$\underbrace{S(\overbrace{a,b,c}^{a^2+b^2+c^2=1}) = \frac{a}{1-c} + i\frac{b}{1-c}}_{\text{Sterographic projection}}$$

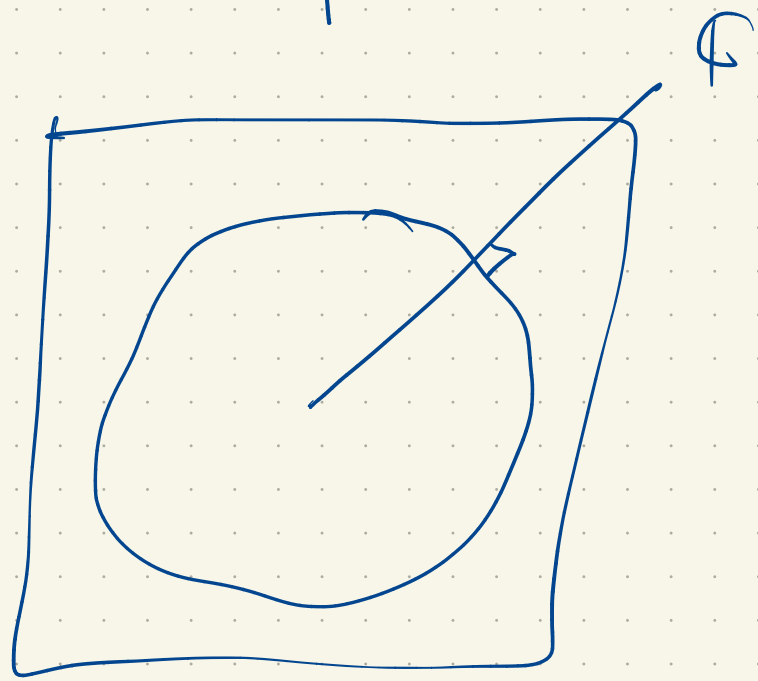
$$f(x) :=$$

4π









Stereographic projection is conformal. (!!)