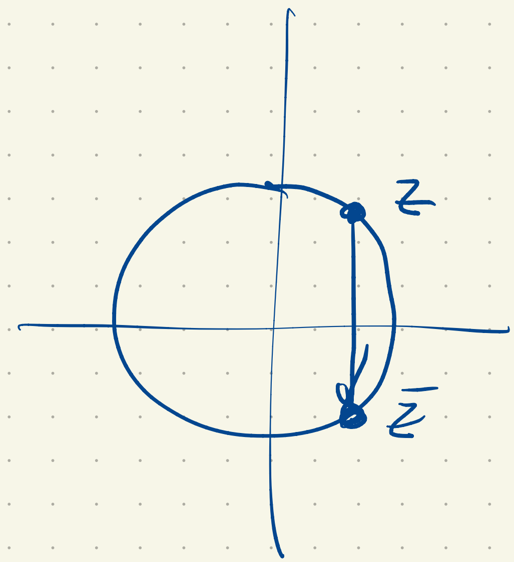
$z \longmapsto z^{-1}$ $\qquad$ $z^{-1} =$ $\qquad$ $z\bar{z} = |z|^2$

$$z \cdot \underbrace{\frac{\bar{z}}{|z|^2}}_{z^{-1}} = 1$$

$$z^{-1} = \frac{\bar{z}}{|z|^2}$$

$Tz = z^{-1}$ $\qquad$ $I(z) = z^{-1}$



1) If $|z| = 1$ $\qquad$ $|I(z)| = 1$

$$I(e^{i\theta}) = e^{-i\theta}$$

2) If $|z| > 1$ $\qquad$ $|I(z)| < 1$

$$\left|\frac{\bar{z}}{|z|^2}\right| = \frac{1}{|z|^2}\cdot|\bar{z}|$$

$$= \frac{|z|}{|z|^2} = \frac{1}{|z|}$$

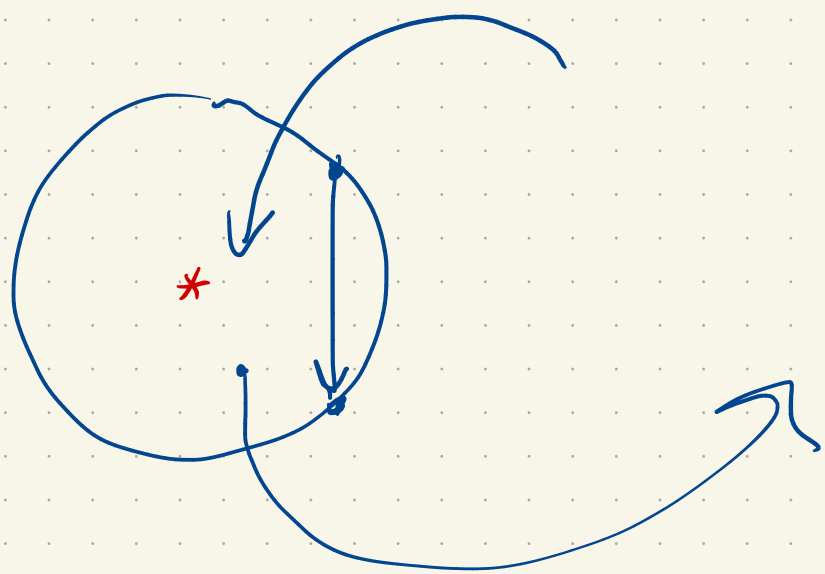
$z = a + ib$

$\bar{z} = a - ib$

$|\bar{z}| = \sqrt{a^2 + (-b)^2}$

$= \sqrt{a^2 + b^2} = |z|$

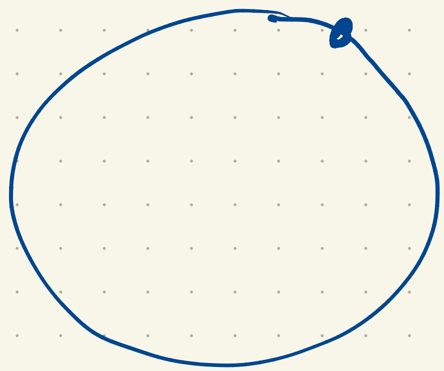
$|z^{-1}| = \dfrac{1}{|z|}$

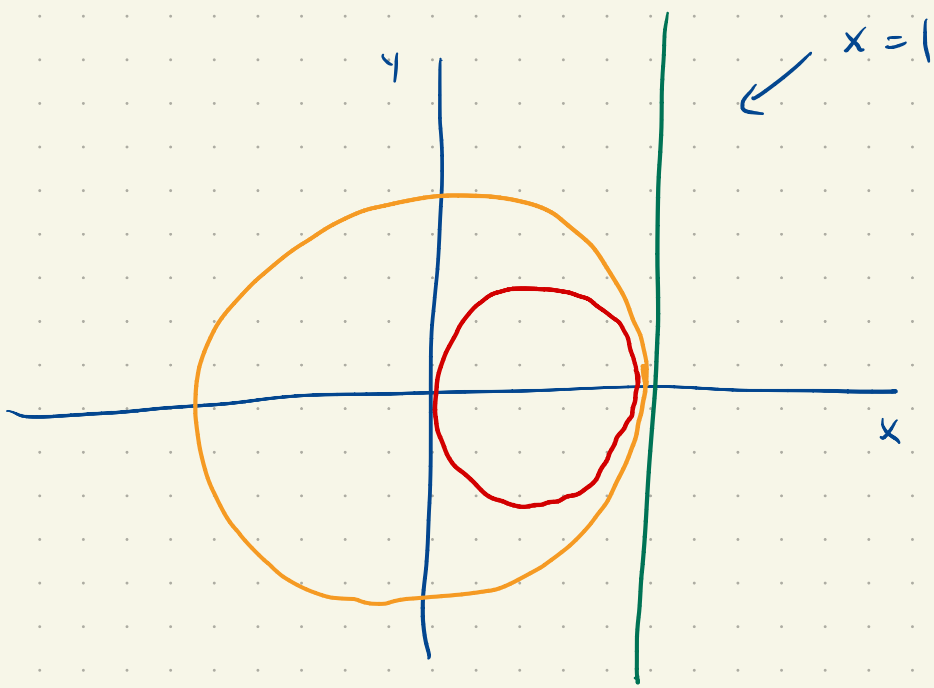


$I(z) = z^{-1} = \dfrac{\bar{z}}{|z|^2}$

$$I(z) = \frac{z}{|z|^2} = \overline{z^{-1}}$$

If $|z| = 1$ $\quad I(z) = z$





I

$$|I(z)| = \frac{1}{|z|}$$

$$z = 1 + is \qquad I(z) = \frac{1}{1+is} = \frac{1-is}{(1+is)(1-is)} = \frac{1-is}{1+s^2}$$

$$\overline{I}(1+is) = \frac{1+is}{1+s^2} = \frac{1}{1+s^2} + i\frac{s}{1+s^2}$$

$$\left(x-\frac{1}{2}\right)^2 + y^2 = \left(\frac{1}{2}\right)^2$$

$$\left(\frac{1}{1+s^2} - \frac{1}{2}\right)^2 + \frac{s^2}{(1+s^2)^2} = \left(\frac{2-(1+s^2)}{2(1+s^2)}\right)^2 + \frac{s^2}{(1+s^2)^2}$$

$$= \left(\frac{1-s^2}{2(1+s^2)}\right)^2 + \frac{2^2 s^2}{2^2(1+s^2)^2}$$

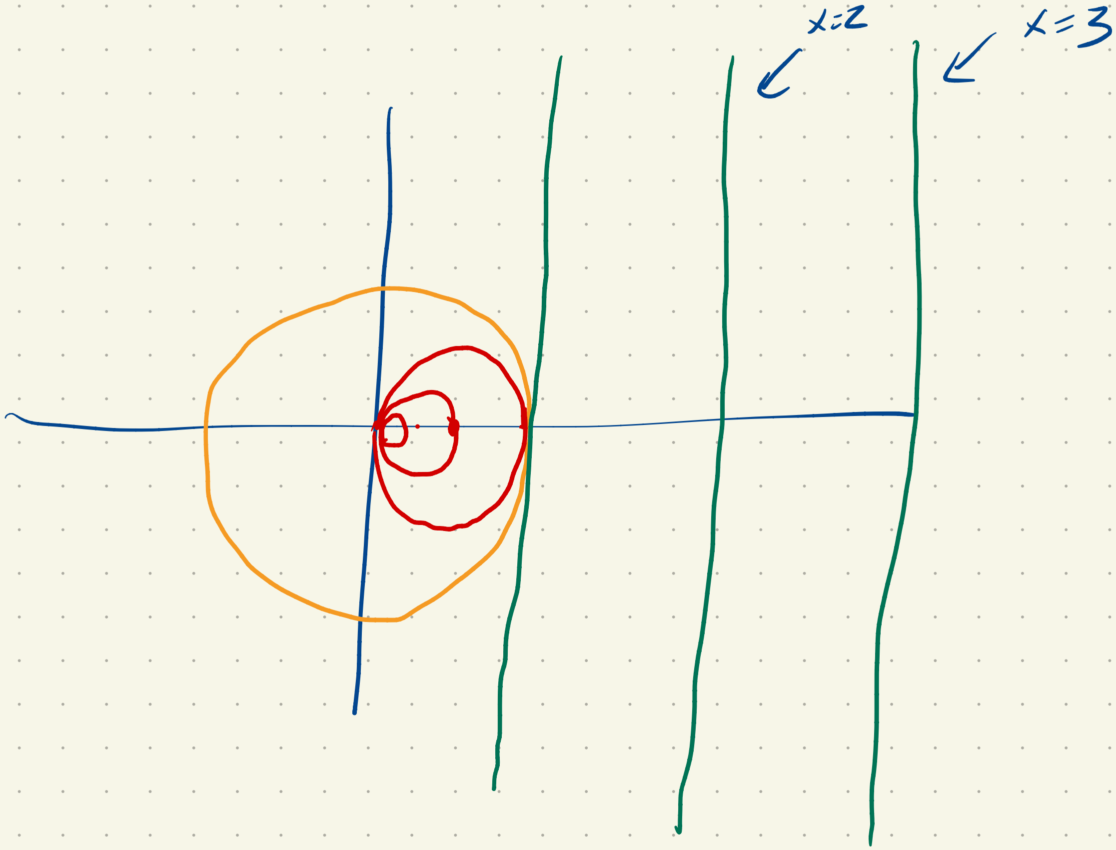
$$= \frac{1-2s^2+s^4+4s^2}{4(1+s^2)^2}$$

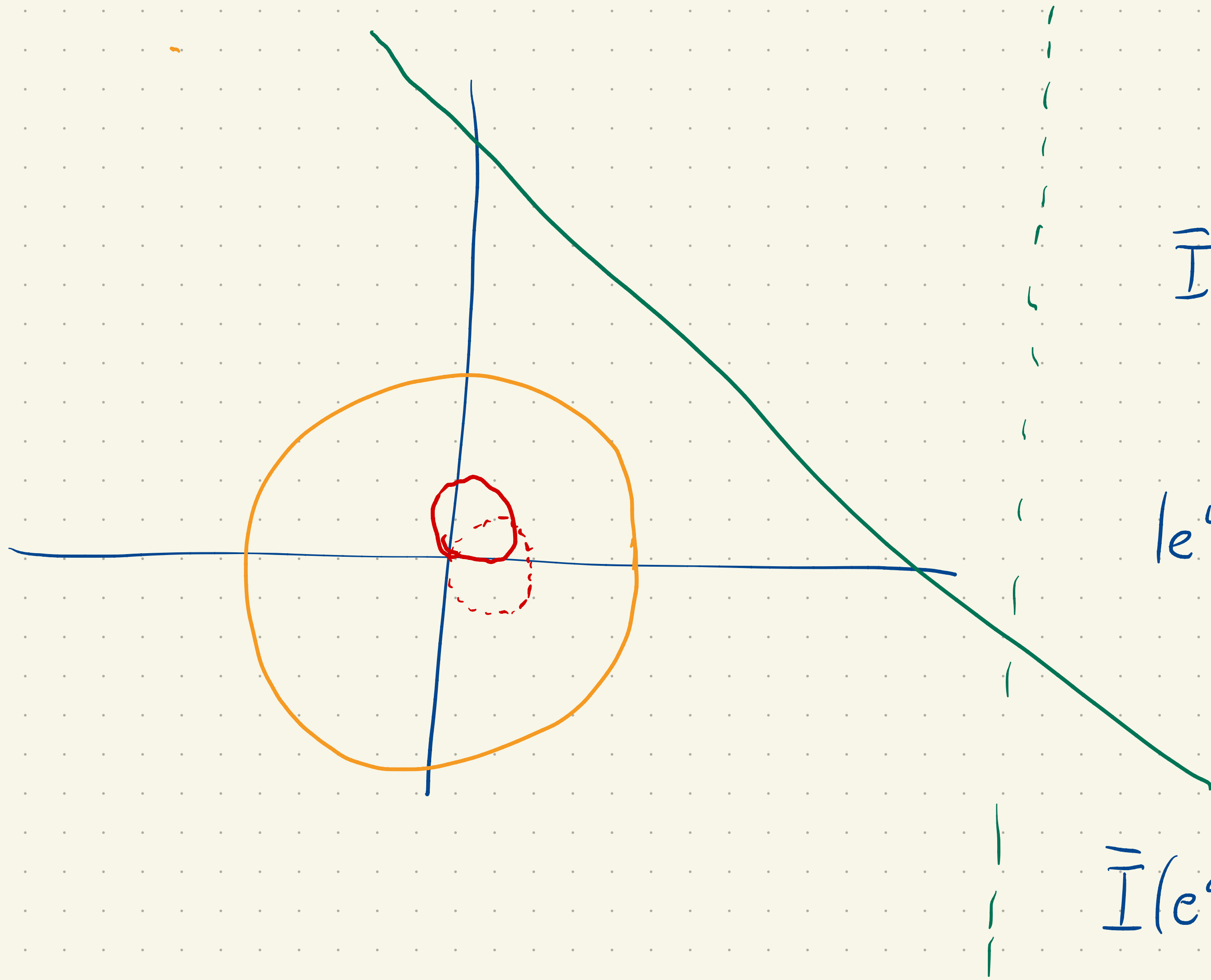
$$= \frac{1 + 2s^2 + s^4}{4\,(1+s^2)^2}$$

$$= \frac{(1+s^2)^2}{4(1+s^2)^2} = \frac{1}{4} = \left(\frac{1}{2}\right)^2$$

$$\bar{I}(az) = \frac{1}{a}\bar{I}(z) \qquad a \in \mathbb{R}$$

$$\bar{I}(az) \quad \frac{az}{|az|^2} = \frac{az}{|a|^2|z|^2} = \frac{az}{a^2|z|^2} = \frac{1}{a}\frac{z}{|z|^2} = \frac{1}{a}\bar{I}(z)$$





$e^{i\theta}$

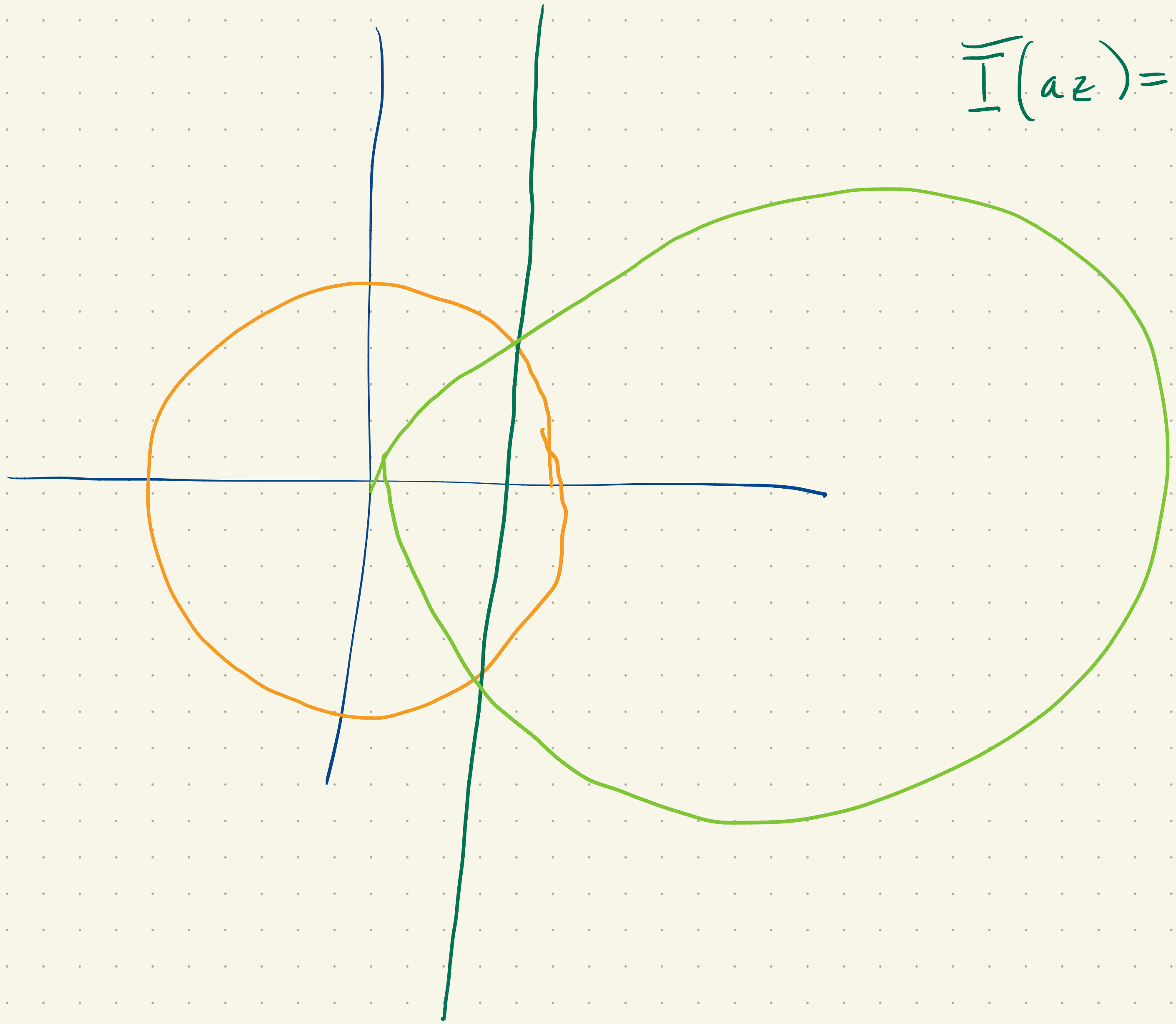
$$\bar{I}(e^{i\theta}z) = \frac{e^{i\theta}z}{|e^{i\theta}z|^2}$$

$|e^{i\theta}| = 1$ $\quad |e^{i\theta}z| = |e^{i\theta}||z|$

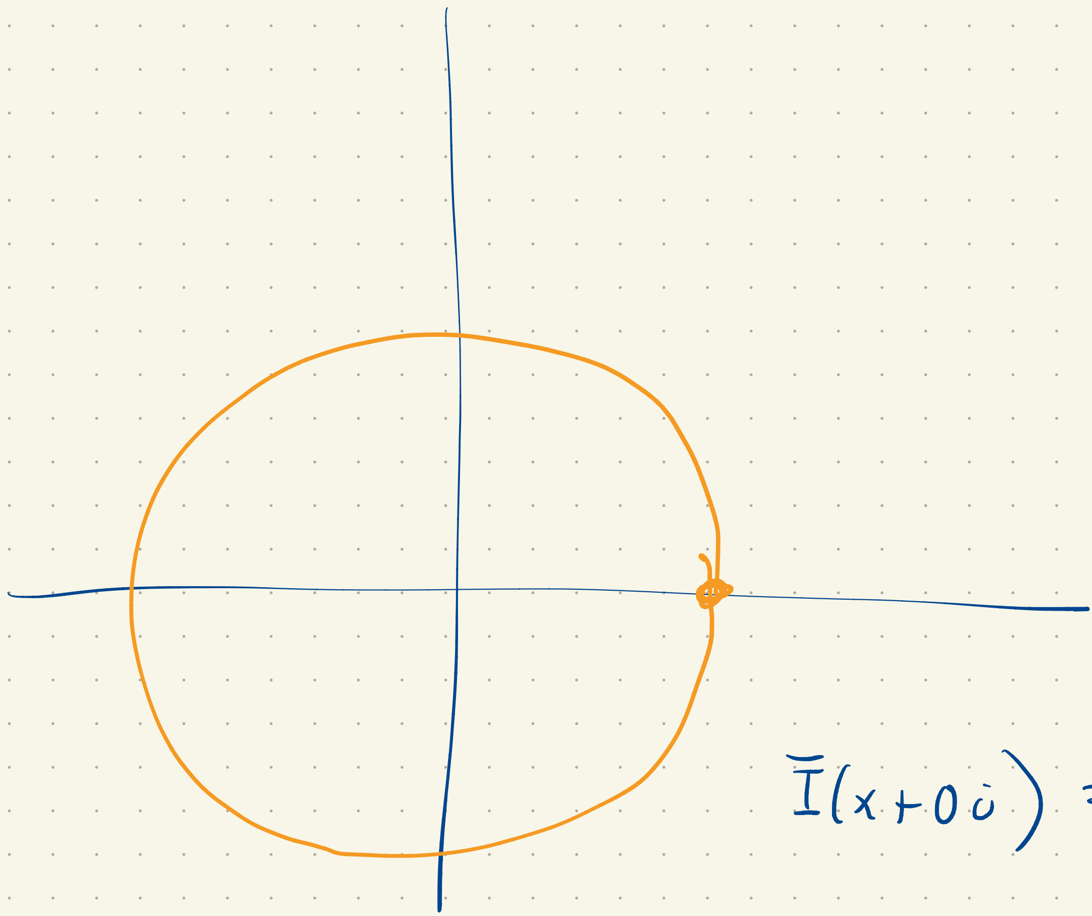
$$= |z|$$

$$\bar{I}(e^{i\theta}z) = e^{i\theta}\frac{z}{|z|^2}$$

$$= e^{i\theta}\,\bar{I}(z)$$



$$\bar{I}(az) = \frac{1}{a}\bar{I}(z)$$



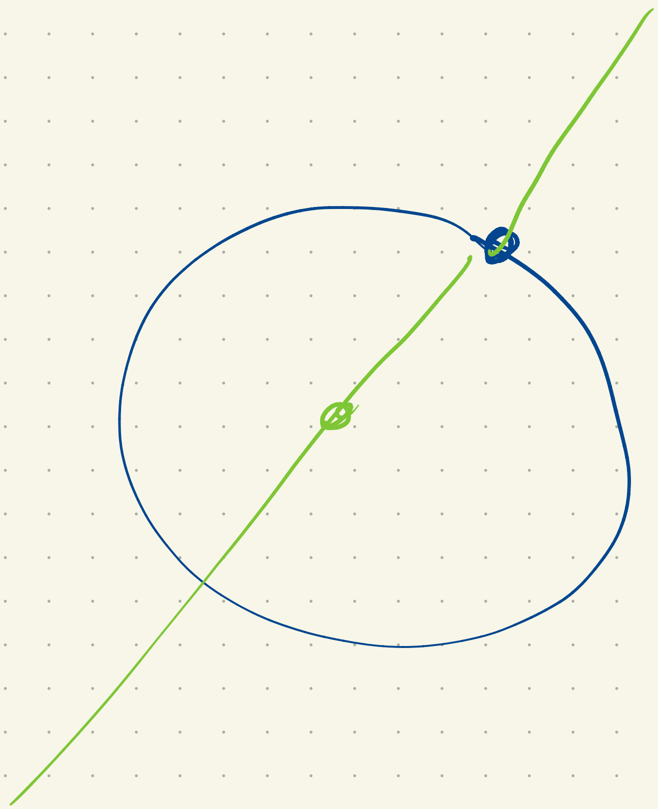
$$\overline{I}(5) =$$

$$\overline{I}(x+0i) = \frac{x+i0}{|x+0i|^2} = \frac{x}{|x|^2}$$

If $x>0$

$$\overline{I}(x) = \frac{1}{x}$$

$ae^{i\theta}$ θ fixed.

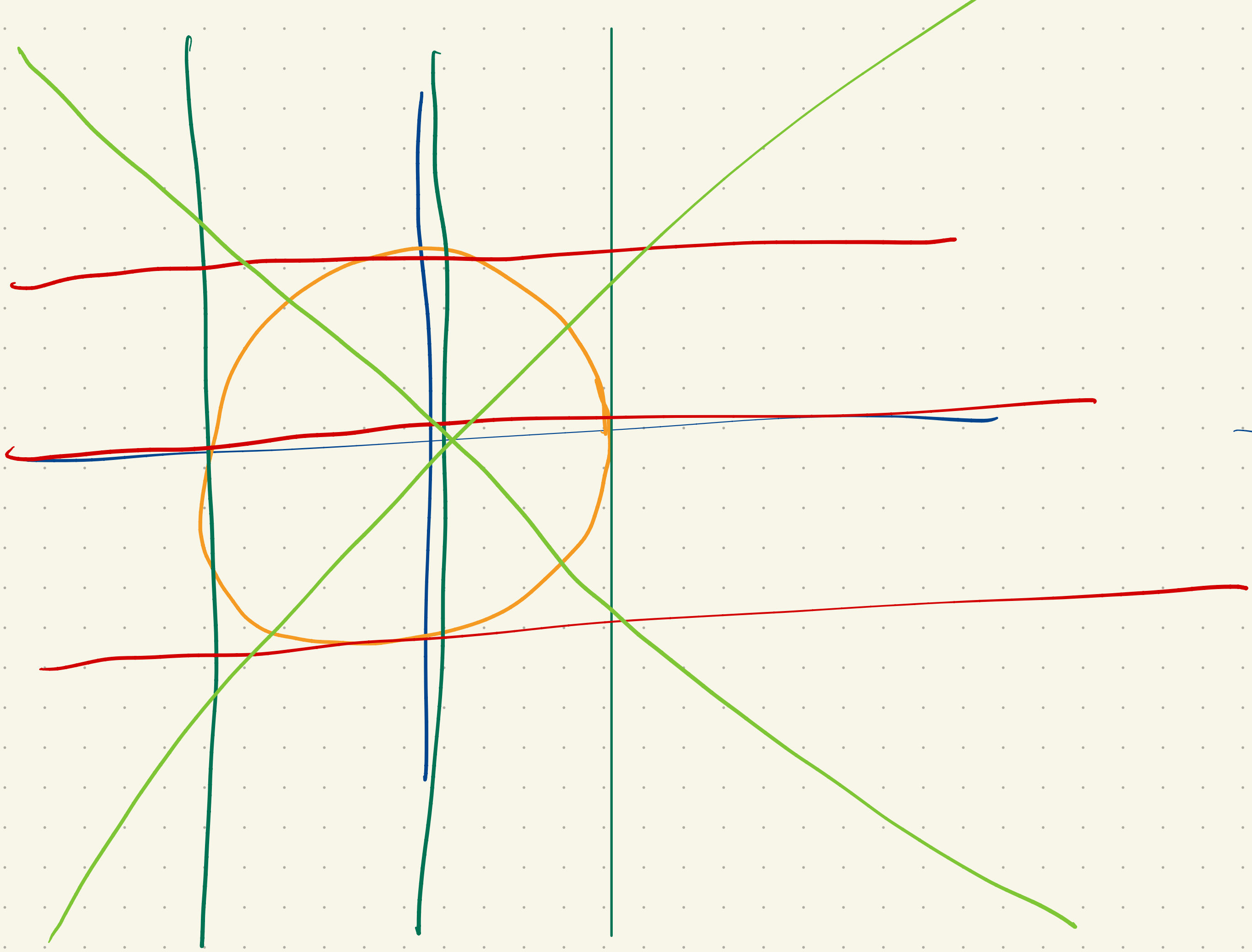
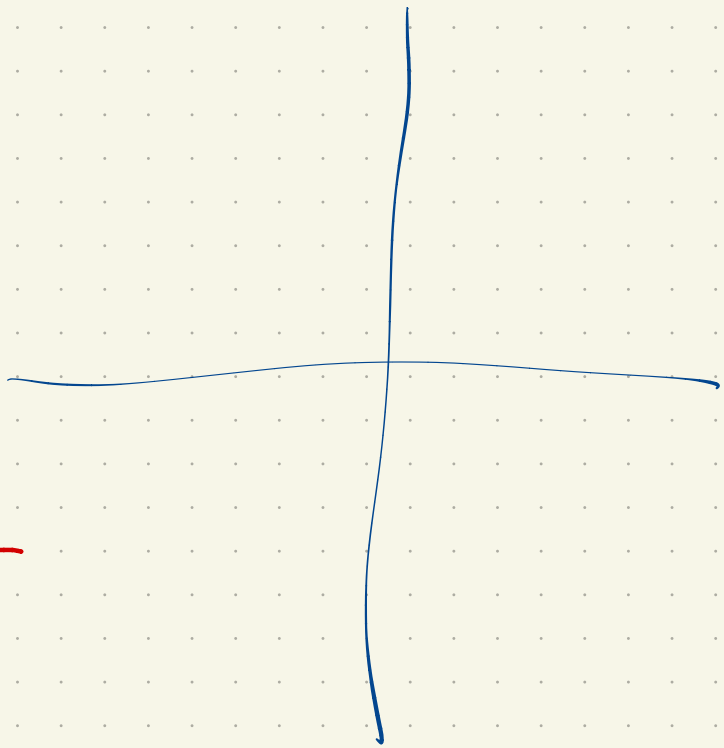


$$I(ae^{i\theta}) = \frac{ae^{i\theta}}{|ae^{i\theta}|^2} = \frac{ae^{i\theta}}{|a|^2}$$

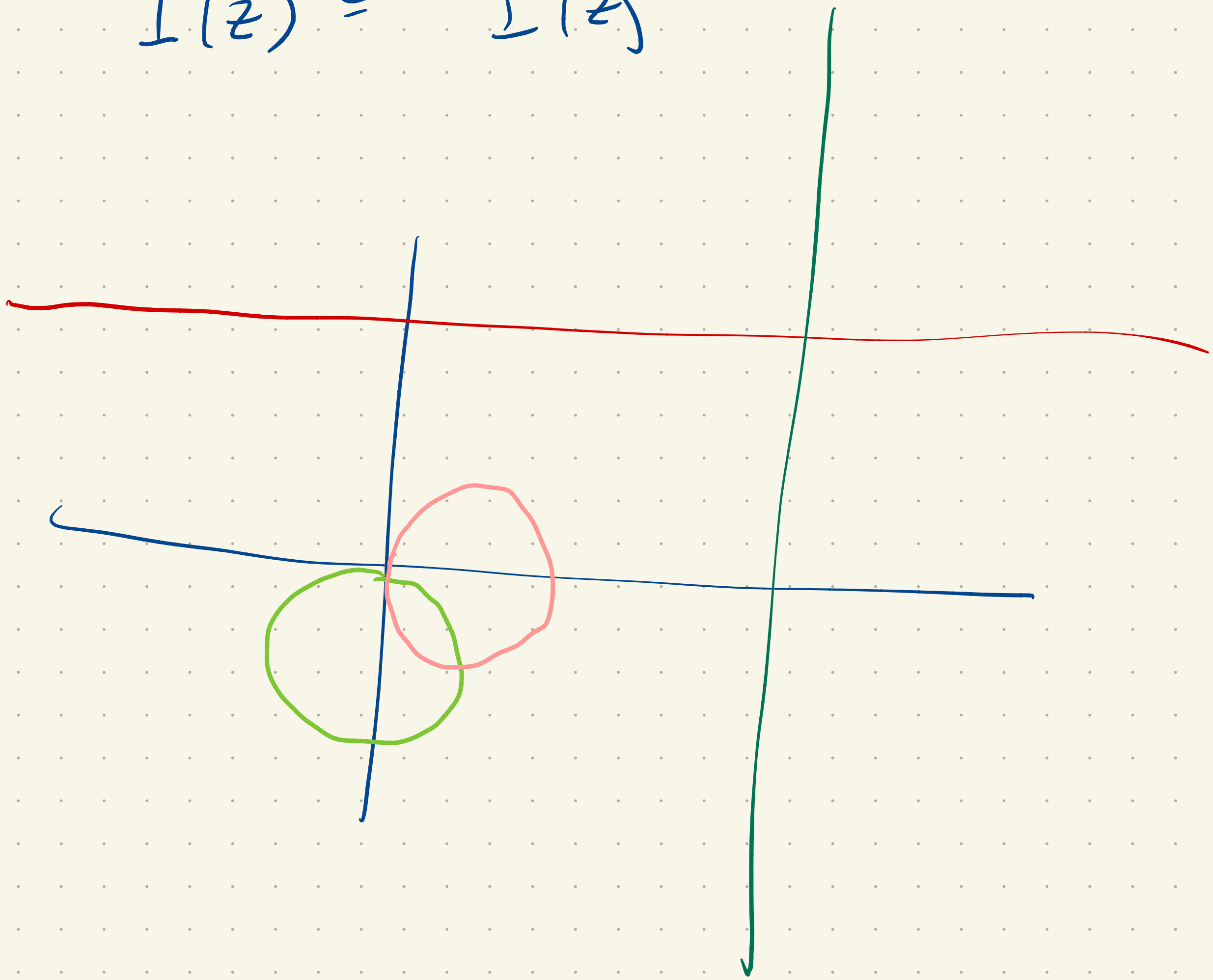
$$= \frac{ae^{i\theta}}{a^2}$$

$\frac{1}{a} e^{i\theta}$

$$= \frac{1}{a} e^{i\theta}$$

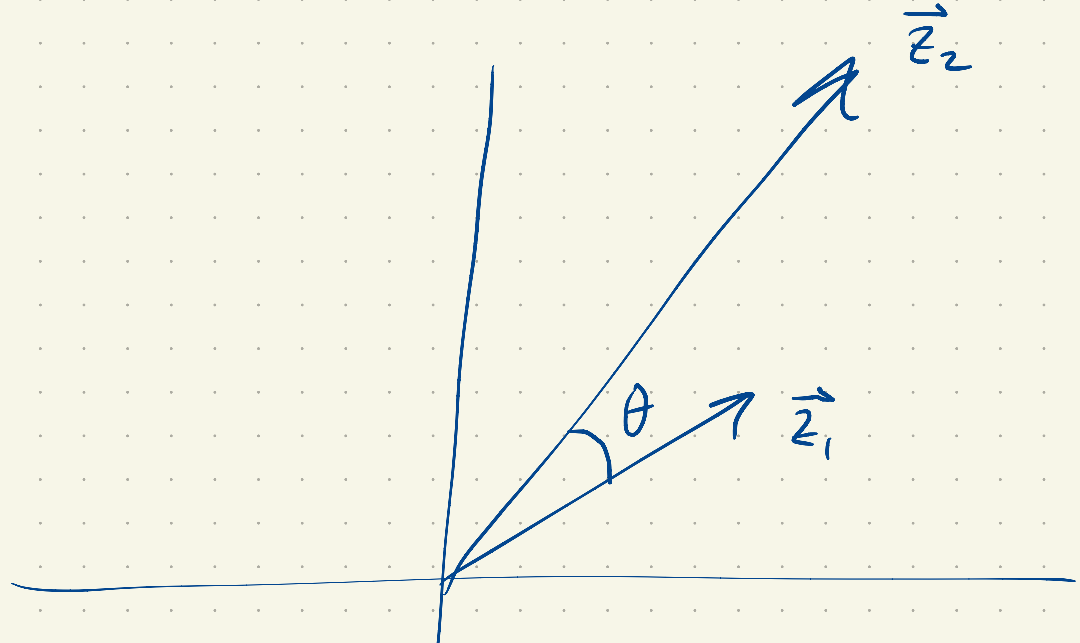
$$I(z) = \overline{\bar{I}(z)}$$



$$z_1 = x_1 + i y_1$$

$$z_2 = x_2 + i y_2$$

$$z_1 \overline{z_2} = (x_1 x_2 + y_1 y_2) + i(-x_1 y_2 + y_1 x_2)$$



$$\vec{z}_1 \cdot \vec{z}_2 = |\vec{z}_1| |\vec{z}_2| \cos\theta$$

$$\downarrow$$

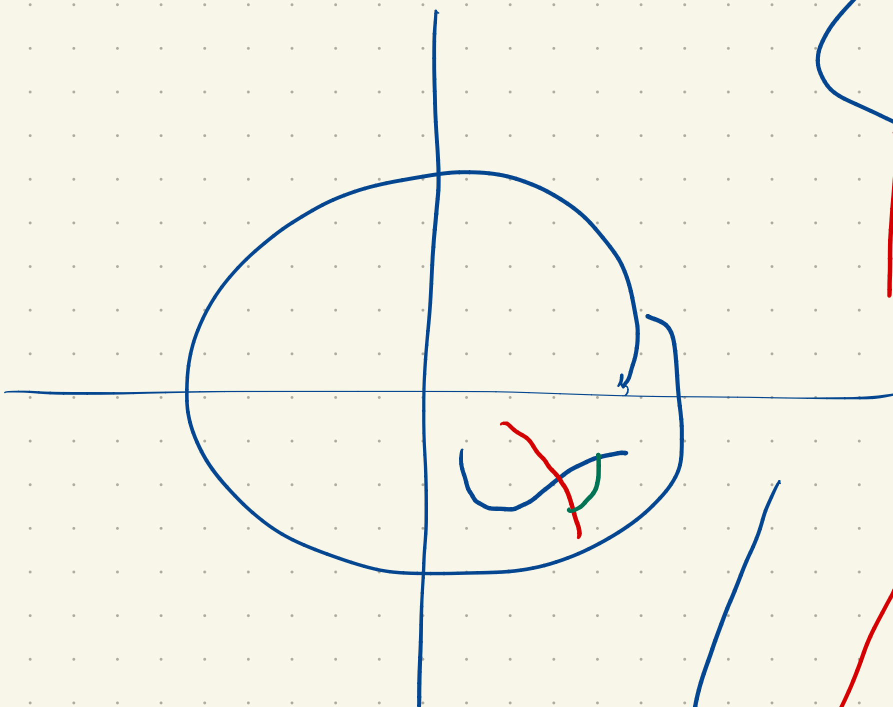
$$x_1 x_2 + y_1 y_2$$

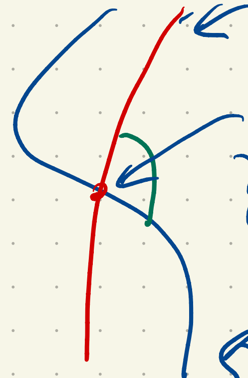
$$\boxed{\mathrm{Re}(z_1 \overline{z_2}) = |z_1| |z_2| \cos\theta}$$

$$\cos\theta = \frac{\mathrm{Re}(z_1 \overline{z_2})}{|z_1| |z_2|}$$

Claim: I preserves angles

"conformal"



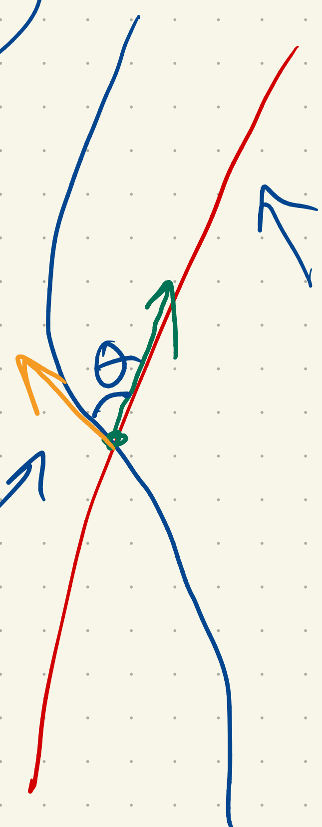


$\gamma_2(t)$

$z \to \frac{1}{z}$

$\underbrace{\gamma_1(0)}_{= \gamma_2(0)}$

$\gamma_1(t)$



$\langle x_1(t), y_1(t) \rangle$

$\langle x_1'(0), y_1'(0) \rangle$

$\langle x_2'(0), y_2'(0) \rangle$

$\gamma_1(t) = x_1(t) + i\, y_1(t)$

$\gamma_1'(t) = x_1'(t) + i\, y_1'(t)$

$$\cos\theta = \frac{\mathrm{Re}\left(\gamma_1'(0)\,\overline{\gamma_2'(0)}\right)}{|\gamma_1'(0)|\,|\gamma_2'(0)|}$$