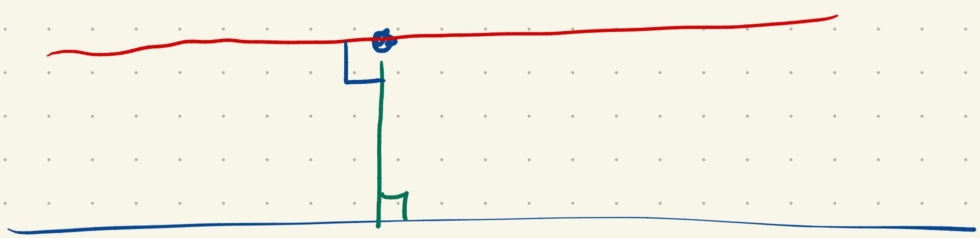
1) If $\angle AEF \neq \angle EFD$ we can assume $\angle EFD$ is smaller

2) But $\angle AEF + \angle FEB = 2\llcorner$

3) So $\angle EFD + \angle FEB$ is less than two right angles.

4) By postulate 5, AB and CD intersect, a contradiction.

I-31 To draw a straight line through a point and parallel to a given line.

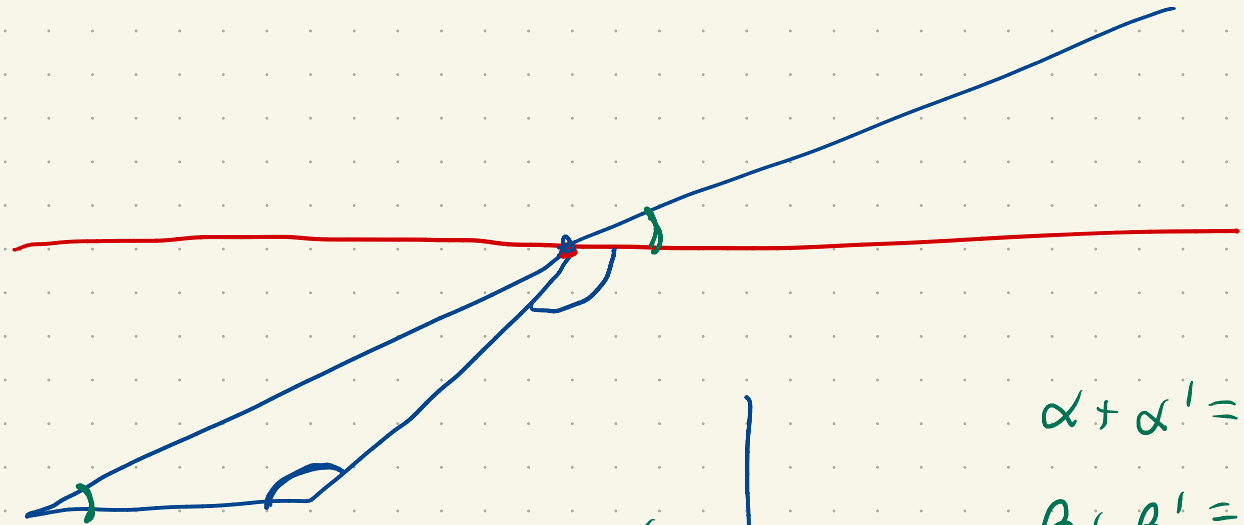


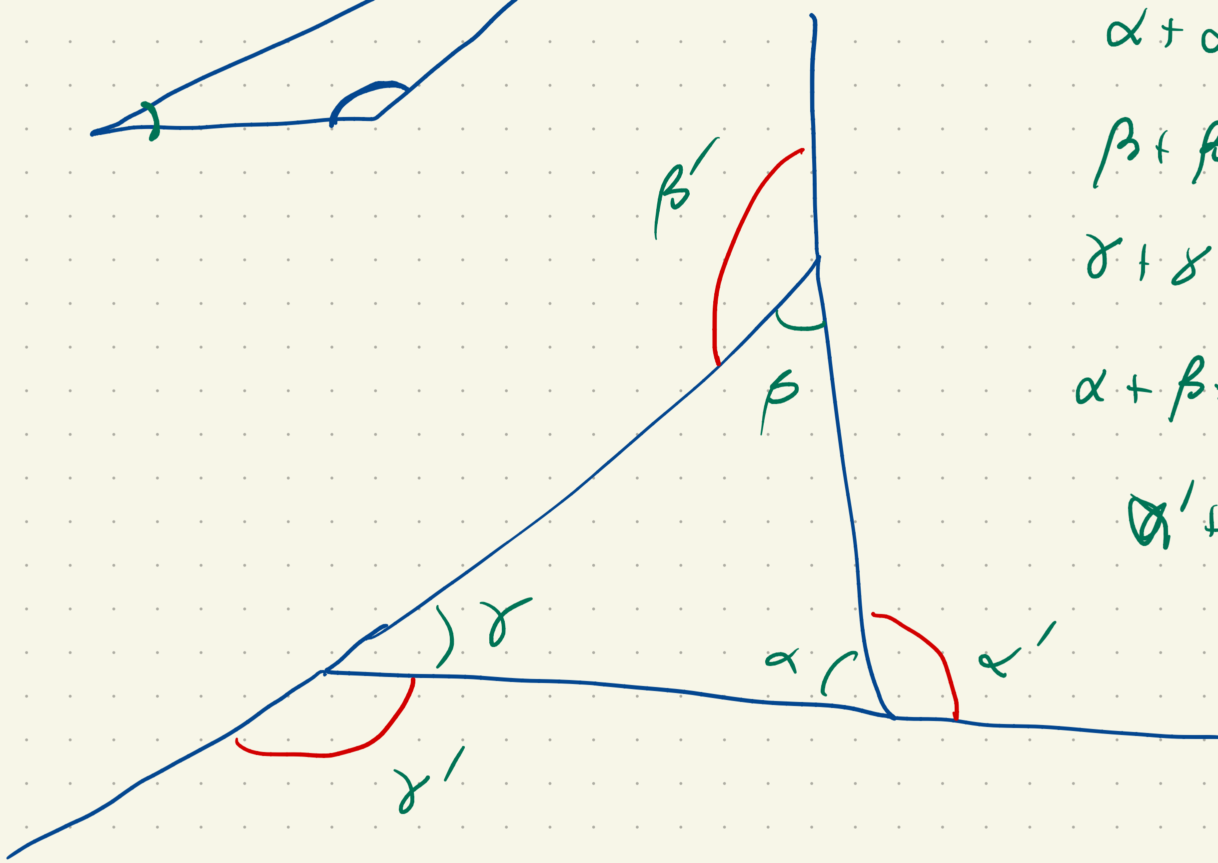
1) drop a perp (I-12)

2) draw a perp line (I-11)

3) parallel! (by I-27

I-32: the sum of interior angles of a Δ is 2 right angles





$$\alpha + \alpha' = 2 \llcorner$$

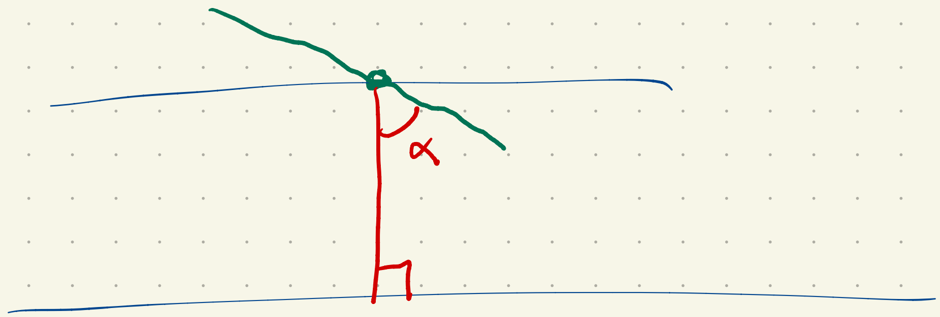
$$\beta + \beta' = 2 \llcorner$$

$$\gamma + \gamma' = 2 \llcorner$$

$$\alpha + \beta + \gamma = 2 \llcorner$$

$$\alpha' + \beta' + \gamma' = 4 \llcorner$$

I-32



$\alpha + \llcorner < 2\llcorner$

green intersects by P5

green is not parallel.

P1–P5: Given a line and a point not on the line there is a unique parallel passing through the point.

Given a line and a point not on the line there is at most one parallel passing through the point.

→ Playfair's Axiom

P1-P5 : ⇒ PF

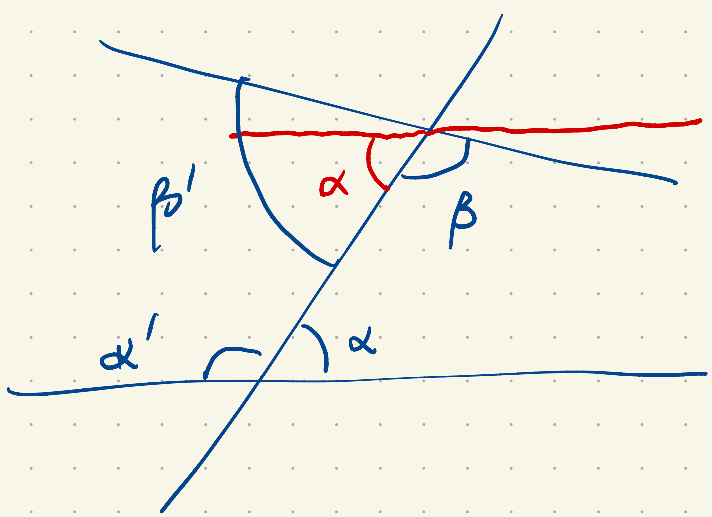
P1-P4 + PF ⇒

If we assume $P1-P4 + PF \Rightarrow P5$ (patched)

$P1-P4 + P5 = PF$

$$\boxed{\begin{array}{l} P1-P4 \\ \qquad P5 \Leftrightarrow PF \end{array}}$$

$P1-P4 + PF \Rightarrow P5$



$\alpha + \beta < 2\llcorner$

1) transfer α to red location

I-2?

2) $\alpha + \beta < 2\llcorner$

$\Rightarrow$ red is a new line.

3) Red is parallel (I-28)

4) PF $\Rightarrow$ blue is not parallel

5) $\alpha' + \beta' > 2\angle$

so intersection is on other side

Other equivalents:

a) Two parallel lines are equidistant (100 BC, Posidonious)

b) If a line intersects one of two parallel lines it intersects the other. (Proclus, 400 AD)

c) Given a Δ one can construct a similar triangle of any size. (Wallis 1600's)

d) The sum of interior angles of a Δ is $2\angle$
Legendre 1700s.

P1-P4

Complex Numbers

We add to $\mathbb{R}$ another number i $\quad i^2=-1$

$\underbrace{\text{Complex numbers}}_{\mathbb{C}}$ $\quad \underbrace{a+ib}_{z}$ $\quad a,b\in\mathbb{R}$

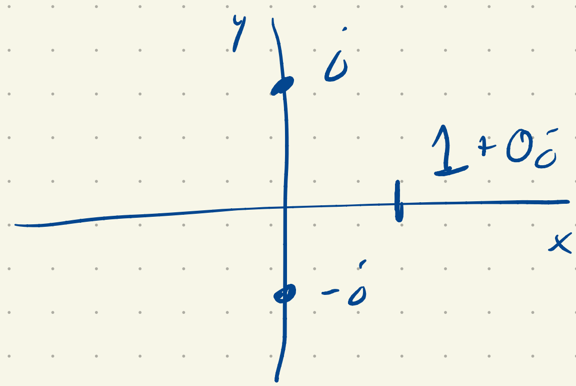
$z = a+ib$

$\mathrm{Re}(z) = a$

$\mathrm{Im}(z) = b$

We identify $\mathbb{R}$ with $\{a+0i : a\in\mathbb{R}\}$

$z = x+iy$



$$(x_1 + i y_1) + (x_2 + i y_2) = (x_1 + x_2) + i(y_1 + y_2)$$

$$\underrightarrow{(x_1 + i y_1) \cdot (x_2 + i y_2)} = (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + x_2 y_1)$$

Exercise:

$$z_1 + z_2 = z_2 + z_1 \qquad (z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$$

$$z_1 z_2 = z_2 z_1 \qquad (z_1 z_2) z_3 = z_1 (z_2 z_3)$$

$$z_1 (z_2 + z_3) = z_1 z_2 + z_1 z_3$$

$$kz = kx + iky \quad \text{if } k \in \mathbb{R}$$