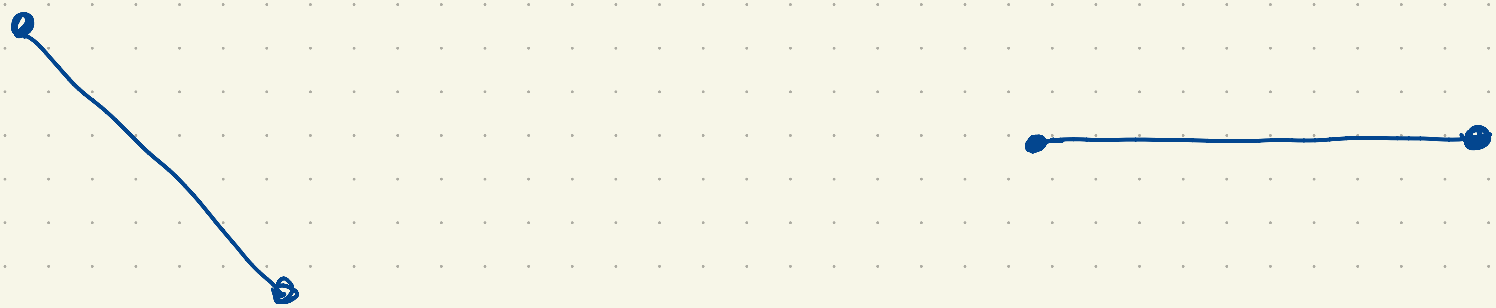
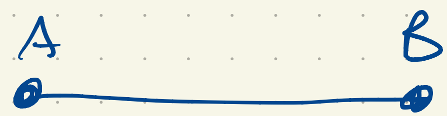
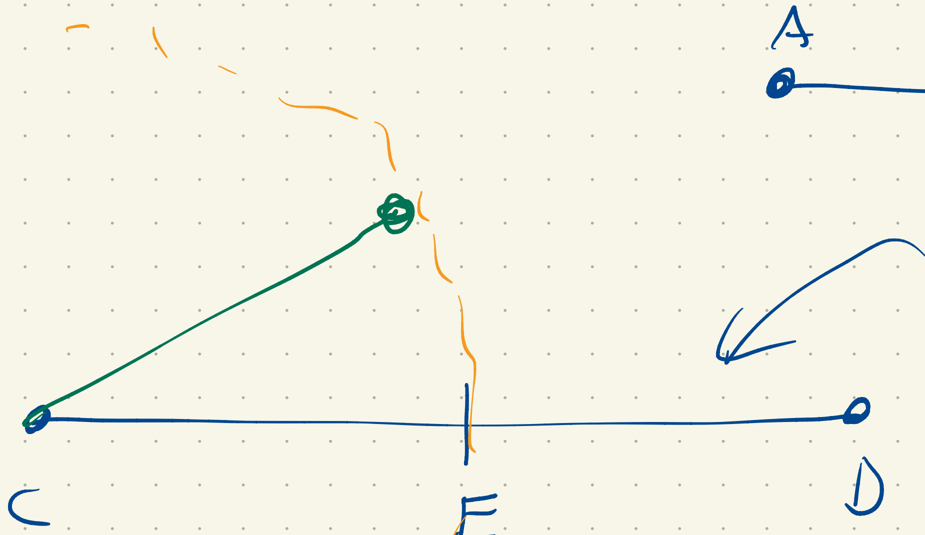


I-3 Subtraction of lines

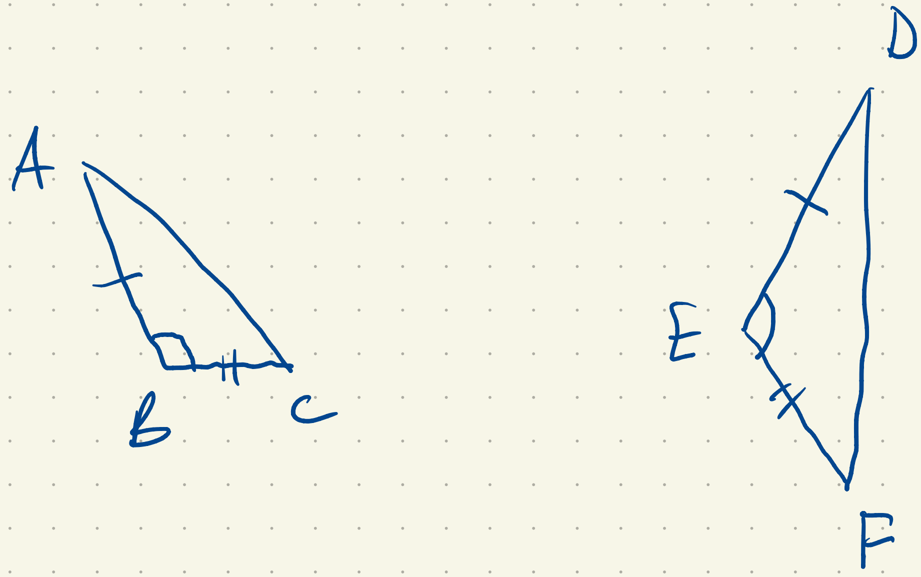




CD > AB

CE = AB and what remains is the result of subtraction.

I-4 (SAS)



$\Rightarrow$

$\measuredangle BAC = \measuredangle EDF$

$\angle BCA = \angle EFD$

$AC = DF$

"superposition" wierdo operation not fleshed out

1) Put B on E so AB lies on DE

2) So A lies on D (length is preserved by superposition?)

3) Then $\measuredangle ABC$ coincides with $\measuredangle DEF$ (angles are preserved by superposition?)

4) So C coincides with F

5) So $CA = FD$, and so on for angles.

Euclid goes out of his way to avoid superposition

(SSS) (AAS, ASA)

I-4, I-8, III-24

(not in others where he could have)

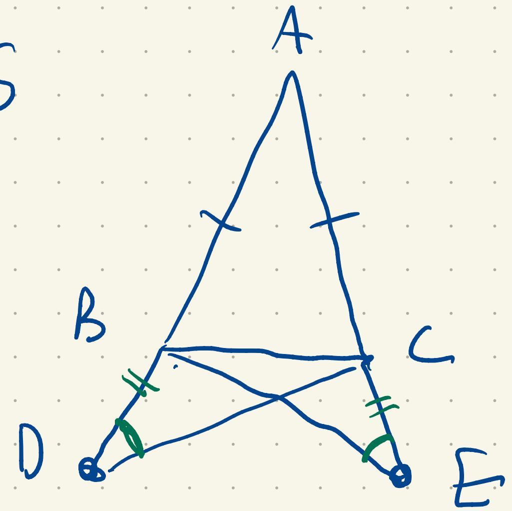
Modern axiomatic geometries take SAS as an axiom

A different modern approach is to fully embrace superposition

We will be interested in operations that preserve "things"

(e.g. length, angle) (isometry) ~

I-5



isoceles triangles have equal angles

$AB = AC$

1) Extend AB to a point D
2) Extend AC to E so that

$CE = BD$

3) Form DC and BE

4) $\triangle ABE = \triangle ACD$ by SAS

5) $\triangle CBD = \triangle BCE$ by SAS

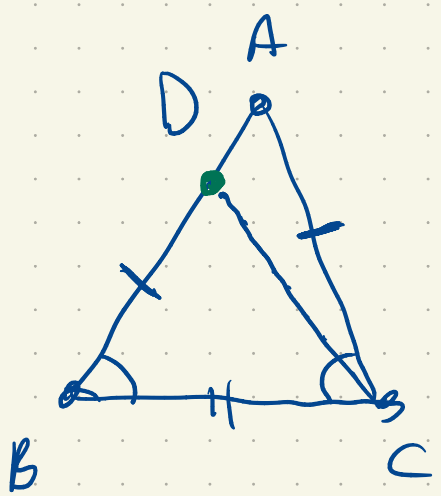
6) $\angle EBC = \angle DCB$

7) $\angle DBC + \angle ABC = 2 ⦜$

8) $\angle ECB + \angle ACB = 2 ⦜$

9) $\angle ABC = \angle ACB$ by subtraction

I-6 if a triangle has two equal angles then it is isoceles



$\Rightarrow AB = AC$

First converse!

(pf. by contradiction)

1) If the length are different then WLOG, $AC < AB$

2) Find D on AB so

$BD = AC$

3) $\angle ABC = \angle ACB$ and

$BC = CB$

so SAS $\Rightarrow$

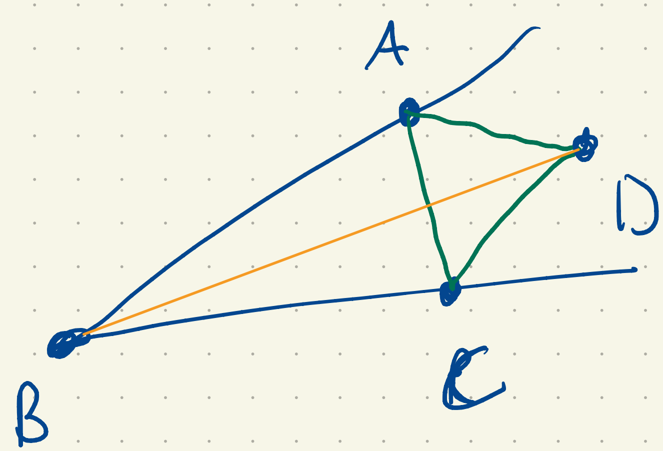
$\Delta DBC = \Delta ACB$

4) So the lesser is equal the whole

I-7 supports:

I-8 SSS via superposition

I-9 Angle bisection

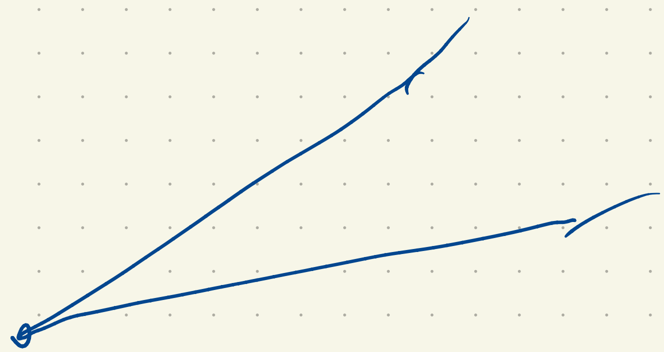


1) arrange so $AB = CB$

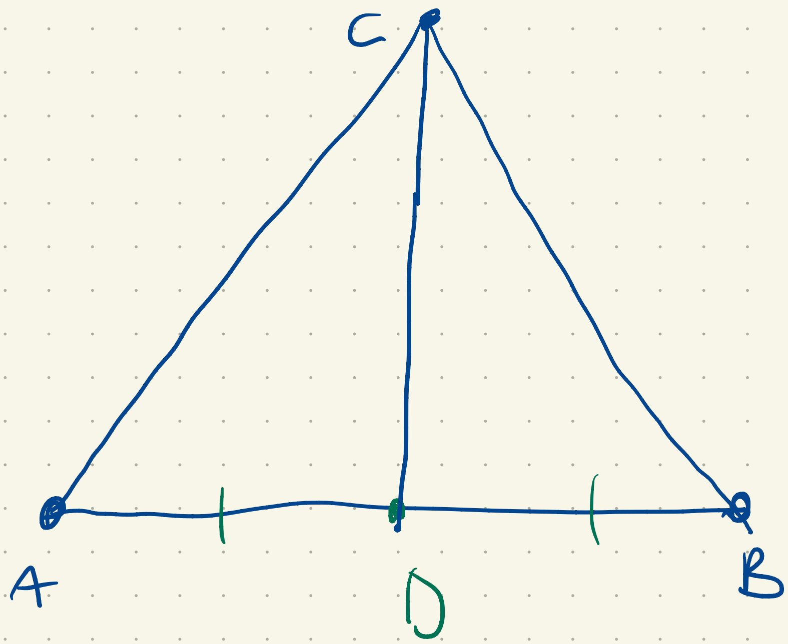
2) Build equilateral $\triangle$ ACD

3) $AB = CB$
$AC = CA$
$BD = DB$

4) $\measuredangle ABD = \measuredangle CBD$
by SSS



I-10 Bisects a line segment

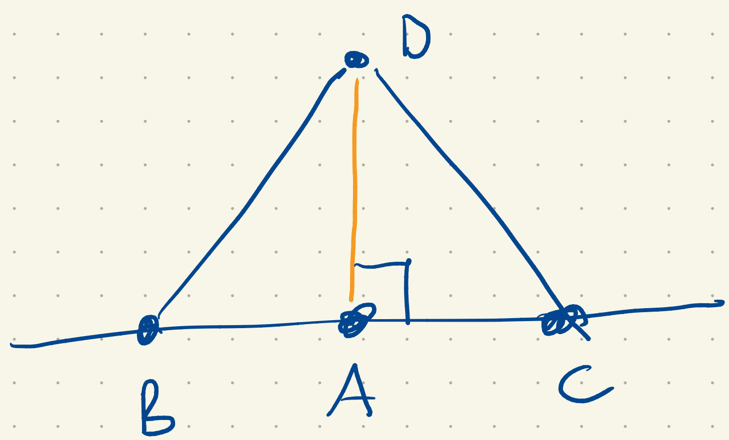


1) Build ΔABC equilateral
2) Bisect $\measuredangle ACB$
3) Extend to D
4) By SAS

   $\Delta ACD = \Delta BCD$

   so $AD = BD$

I-11 Extending a perpendicular



1) Find points B, C with

   $AB = AC$

2) Make an equilateral Δ

   ΔBDC

3) By SSS $\measuredangle DAB = \measuredangle DAC$