Some Euclid for Context

Elements ~ 300 BC

13 books
- plane geometry
- number theory (GCDs!)
- solid geometry

We'll only look at book I

Online: David Joyce (on website)

Book I: basic plane geometry ending with Pythagorean Thm.

- 23 definitions
- 5 axioms (postulates)
- 5 "common notions" (too basic to even be postulates)
- 48 propositions, the heart

By modern standards, the definitions are loosy goosy

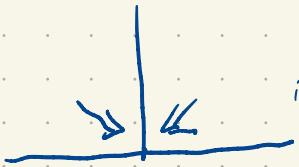
1) A point is that which has no part
2) A line is breadthless width. Hmm. (not straight!)
3) The ends of lines are points.
4) A straight line is a line that lies evenly on itself.
   Yikes. Nobody knows what this means.

As mathematics matured we gave up on these. In modern axiomatic geometry points and lines would be undefined concepts and we'd only have axioms for how they work.

8) A plane angle is the inclination to one another of two lines in a plane which meet one another and do not lie in a straight line.

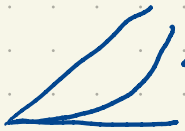
9) Rectilinear

10)



This has implicit: angles have a notion of equality.

Later: also comparison.



15) A circle is a line, and it contains a point so that all straight lines from the point to other points on the circle are equal,

Again with the equality. (also: inside!)

Common Notions: (cover first)

1) $A = B, B = C \Rightarrow A = C$ (equality is transitive)

2) $A = B, C = D \Rightarrow A + C = B + D$

3) ditto for subtraction

4) [coincide $\Rightarrow$ equals]

5) Whole > part. Hmm.

Postulates:

1) Given two points, there is a line incident to both.

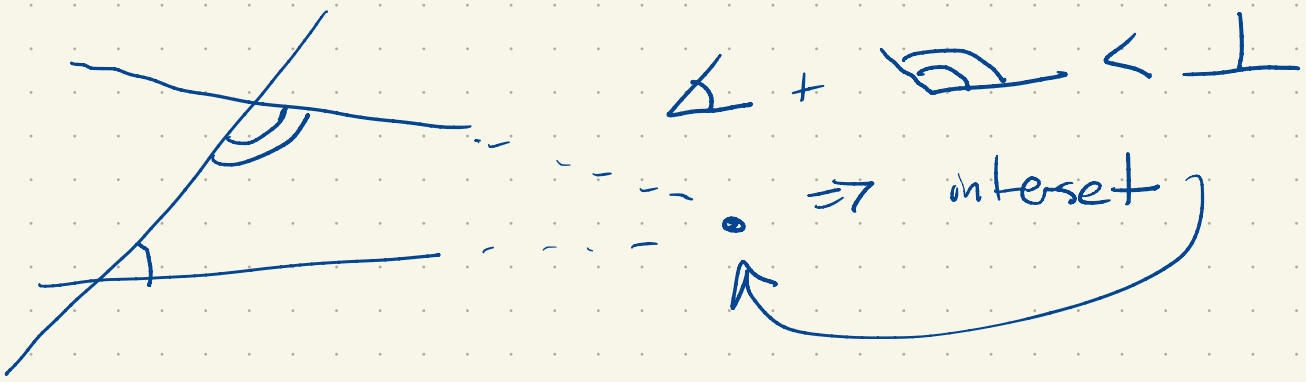
2) A straight line can be extended.

3) Given a point A and another point B, (collapse!)
there is a circle with center A incident to B

4) All right angles are equal

5) If a straight line lies on two straight lines makes the interior angles on one side less than two right angles, the two lines if extended indefinitely

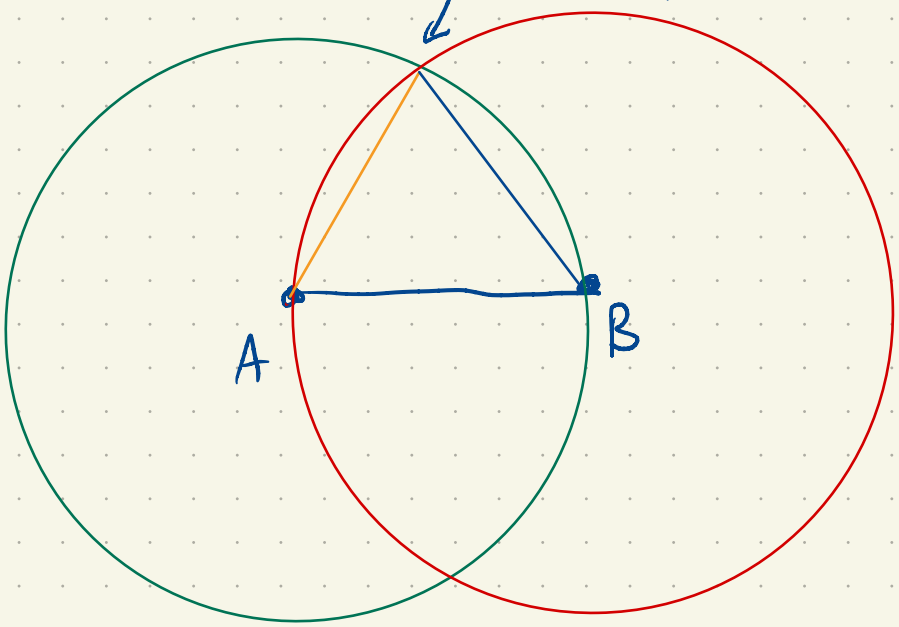
intersect on the same side



(It's a doozy!)

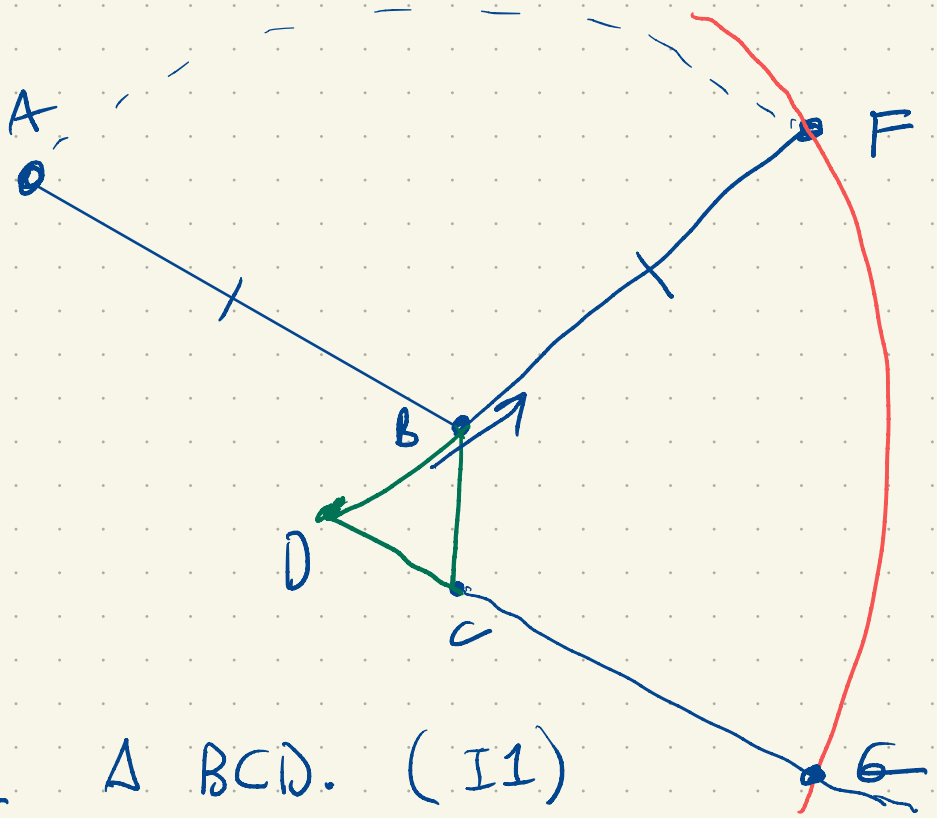
Propositions

I 1 construction of equilateral triangles



$AC = AB, \quad BC = AB \Rightarrow AC = CB$

I2 Given a line (segment) and another point, you can make a new line segment starting at the point that equals the original. (equal must be in terms of length)



1) Make $\triangle BCD$. (I1)

2) Extend DB.

3) Make circle of radius AB centred at B.

4) Let F be point of intersection (oops!)

5) New circle: D centre, radius DF.

6) Eted DC so it intesects at G

7) DF = DG and DB = DC so AB = BF = CG.