

1. Suppose

$$T(z) = e^{i\theta} \frac{z - p}{1 - z\bar{p}}$$

where $\theta \in \mathbb{R}$ and p is in the open unit disc D . We showed in class that every hyperbolic transformation has this form. The goal of this problem is to establish the converse.

- a) Show that if $|z| < 1$ then $|T(z)| < 1$. You may find it easier to work with $|Tz|^2$, which is fine.
- b) Show that T^{-1} can be written in the same form.
- c) Conclude that every Möbius transformation of this form is a hyperbolic transformation.