1. Compute the linearization of $f(x) = 1/x$ at $x = 2$.

$f'(x) = -1/x^2$

$f(2) = 1/2$

$f'(2) = -1/4$

$L(x) = f(a) + f'(a)(x-a)$

$L(x) = \frac{1}{2} - \frac{1}{4}(x-2)$

$0.5 - 0.75(x-2)$

2. Use your linearization to estimate $1/3$.

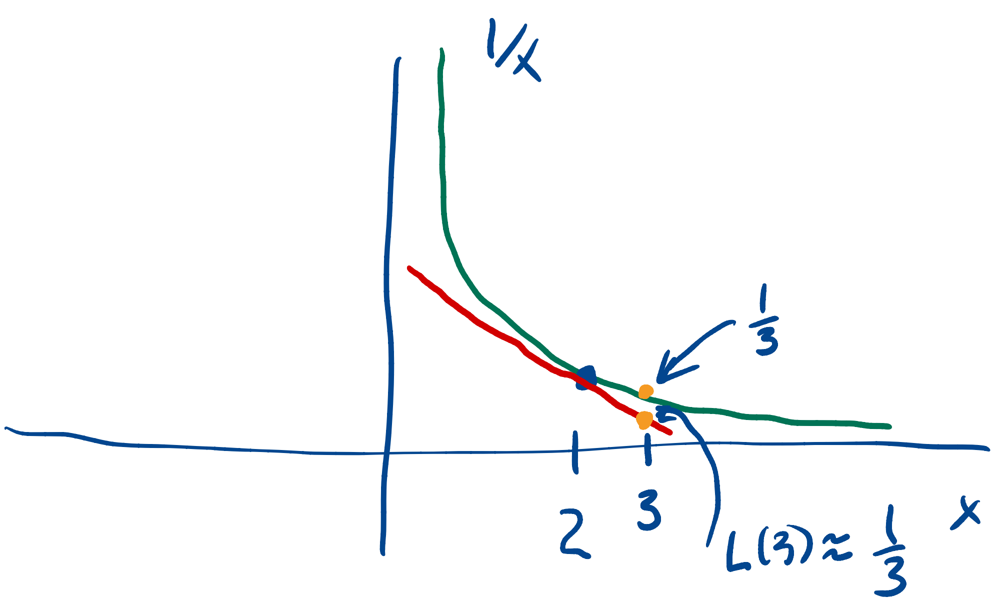
$L(x) \approx \frac{1}{x}$ if x is near 2

$L(3) \approx \frac{1}{3}$

$L(3) = \frac{1}{2} - \frac{1}{4}(3-2)$

$= \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

3. Draw a graph that illustrates the computation you just did. Then use the graph to determine if your estimate for $1/3$ is an underestimate or an overestimate.

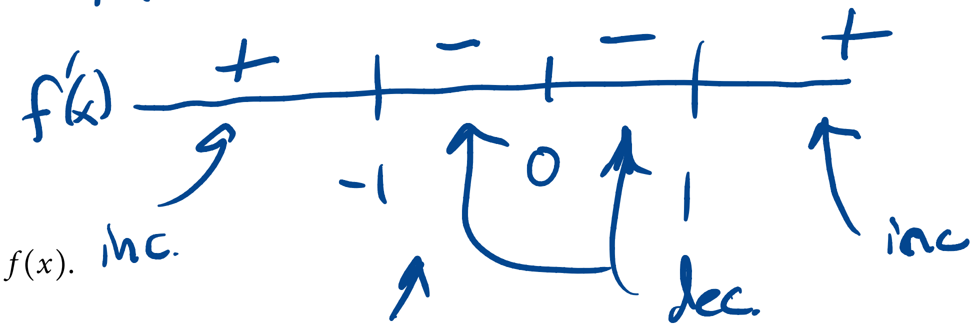


$(-\infty,-1),\ (1,\infty)$

The problems on this page refer to the function $f(x) = \dfrac{1}{x} + x$.

4. On what intervals is the function $\underbrace{\text{increasing?}}_{f'(x)>0}$ $\underbrace{\text{Decreasing?}}_{f'(x)<0}$

$$f'(x) = \frac{-1}{x^2} + 1 = \frac{x^2-1}{x^2}$$



5. Find the critical points of $f(x)$.

$x = -1, 1$

$\hookrightarrow f'(x) = 0$

6. Use the first derivative test to classify the only positive critical point as a local min/max/neither.

local min

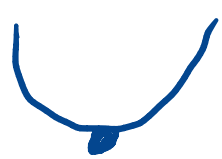
7. Use the second derivative test to classify the only positive critical point as a local min/max if this is possible

$$f'(x) = \frac{-1}{x^2} + 1$$

$$f''(x) = 2x^{-3} = \frac{2}{x^3}$$

$$f''(1) = 2 > 0$$

concave up at $x=1$



local min

8. A circular metal plate is being heated in an oven. The radius of the plate is increasing at a rate of 0.01 cm/min when the radius is 50cm. How fast is the area of the plate increasing?

cm^2/min

radius: r, cm

area of plate: A, cm^2

$$\frac{dr}{dt} = 0.01 \ \frac{\text{cm}}{\text{min}} \quad \text{when} \quad r = 50 \text{ cm}$$

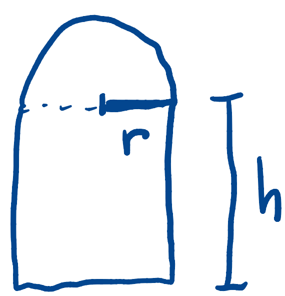
Want $\frac{dA}{dt}$.

$$A = \pi r^2$$

$$\frac{dA}{dt} = \pi \cdot 2r \frac{dr}{dt} = 2\pi \cdot 50 \cdot \frac{1}{100}$$
$$= \pi \ \text{cm}^2/\text{min}$$

9. A Norman window is has a rectangular base and a semi-circle on top. What dimensions of the window minimize the perimeter if the area of the window is to be 4 ft^2.

Applied Opt.



Optimize perimeter.

Area: $2rh + \frac{1}{2}\pi r^2$

Perimeter $P = 2h + 2r + \pi r$

$$2rh + \frac{1}{2}\pi r^2 = 4$$
$$2rh = 4 - \frac{1}{2}\pi r^2$$

$$h = \frac{4}{2r} - \frac{1}{4}\pi r$$

$$= \frac{2}{r} - \frac{1}{4}\pi r = \frac{8 - \pi r^2}{4r}$$

$$P = 2h + 2r + \pi r$$

$$P = \frac{8 - \pi r^2}{2r} + 2r + \pi r$$

$$= \frac{4}{r} - \frac{\pi}{2} r + 2r + \pi r$$

$$= \frac{4}{r} + \left(2 + \frac{\pi}{2}\right) r$$

$$\frac{dP}{dr} = -\frac{4}{r^2} + \left(2 + \frac{\pi}{2}\right)$$

$$-\frac{4}{r^2} + \left(2 + \frac{\pi}{2}\right) = 0$$

$$r^2 = \frac{4}{2 + \pi/2} = \frac{8}{4 + \pi}$$

$$r = \sqrt{\frac{8}{4 + \pi}} \approx 1.058 \text{ ft}$$

$$h = \frac{8 - \pi r^2}{4r}$$

substitute $r = \sqrt{\frac{8}{4+\pi}}$ into

$$h = \sqrt{\frac{8}{4+\pi}}$$

(!)

10. The volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$ where r is the radius of the base of the cone and h is the height of the cone. Use a differential to estimate the change in volume of the cone if the height is fixed at 9 feet and the radius changes from 5 feet to 5.5 feet.

$$dV = \frac{2}{3}\pi r\,dr\,h$$

$$dV = \frac{2}{3}\pi \cdot 9 \cdot 5 \cdot \frac{1}{2} = 15\pi$$

11. Compute $\lim_{x\to 0} \dfrac{\sec(x) - 1}{x^2}$

$$\lim_{x\to 0} \frac{\sec(x)-1}{x^2} \overset{\frac{0}{0}}{=} \lim_{x\to 0} \frac{\sec(x)\tan(x)}{2x}$$

$$\overset{\frac{0}{0}}{=} \lim_{x\to 0} \frac{\sec(x)\tan^2(x) + \sec^3(x)}{2}$$

$$= \frac{0+1}{2} = \frac{1}{2}$$

12. Consider the curve defined implicitly by

$$x^4 + y^4 = 2.$$

a. Show that the point $(1,1)$ lies on this curve.

$$1^4 + 1^4 = 2$$

b. Find the slope of the tangent line to the curve at this point.

$$4x^3 + 4y^3 y' = 0$$

$$y' = \frac{-x^3}{y^3}$$

$$y' = \frac{-1^3}{1^3} = \boxed{-1}$$