

1. Compute $\int_0^{\pi/2} \cos^3(x) \sin(x) dx$

$$u = \cos(x) \quad \sin(x) dx = -du$$

$$du = -\sin(x) dx$$

$$x = \frac{\pi}{2} \Rightarrow u = \cos(\pi/2) = 0$$

$$x = 0 \Rightarrow u = \cos(0) = 1$$

$$\int_1^0 u^3 du = \int_0^1 u^3 du$$

$$= \left. \frac{u^4}{4} \right|_0^1 = \frac{1}{4}$$

2. Compute $\int \cos(x) \sin(\sin(x)) dx$

$$u = \sin(x)$$

$$du = \cos(x) dx$$

$$\int \sin(u) du = -\cos(u)$$

$$= -\cos(\sin(x))$$

3. Compute $\int \frac{1}{9+x^2} dx$

$$\int \frac{1}{9+x^2} dx = \int \frac{1}{9} \frac{1}{1+(x/3)^2} dx$$

$$u = x/3$$

$$du = \frac{1}{3} dx$$

$$3 du = dx$$

$$= \frac{3}{9} \int \frac{1}{1+u^2} du = \frac{1}{3} \arctan(u)$$

$$= \frac{1}{3} \arctan(x/3)$$

4. Compute $\int \sqrt{x}(x^4 + x) dx$

$$\int x^{9/2} + x^{3/2} dx = \frac{2}{11} x^{11/2} + \frac{2}{5} x^{5/2} + C$$

5. Compute $\int x\sqrt{x-1} dx$

$$u = x - 1 \\ du = dx$$

$$\begin{aligned} \int (u+1)\sqrt{u} du &= \int u^{3/2} + u^{1/2} du = \frac{2}{5} u^{5/2} + \frac{2}{3} u^{3/2} \\ &= \frac{2}{5} (x-1) + \frac{2}{3} (x-1)^{3/2} \end{aligned}$$

6. Compute $\int_1^3 \frac{(\ln(x))^3}{x} dx$

$$u = \ln(x) \\ du = \frac{1}{x} dx$$

$$x = 1 \Rightarrow u = \ln(1) = 0$$

$$x = 3 \Rightarrow u = \ln(3)$$

$$\int_0^{\ln(3)} u^3 du = \left. \frac{u^4}{4} \right|_0^{\ln(3)} = \frac{(\ln(3))^4}{4}$$

7. Compute $\frac{d}{dx} [x \ln(x) - x]$. Then compute $\int s^2 \ln(s^3) ds$

$$\frac{d}{dx} (x \ln(x) - x) = 1 \cdot \ln(x) + \frac{x}{x} - 1 = \ln(x)$$

$$u = s^3, \quad du = 3s^2 ds, \quad \frac{1}{3} du = s^2 ds$$

$$\int s^2 \ln(s^3) ds = \int \frac{1}{3} \ln(u) du = \frac{1}{3} [u \ln(u) - u]$$

$$= \frac{1}{3} [s^3 \ln(s^3) - s^3]$$

8. Compute $\int \cot(\theta) d\theta$

$$\int \frac{\cos \theta}{\sin \theta} d\theta \quad \begin{array}{l} u = \sin \theta \\ du = \cos \theta d\theta \end{array}$$

$$\int \frac{du}{u} = \ln(|u|) = \ln(|\sin \theta|) + C$$

9. Compute $\int x(x+1)^{1/4} dx$

$$\begin{array}{l} u = x+1 \\ du = dx \end{array}$$

$$\int (u-1)u^{1/4} du = \int u^{5/4} - u^{1/4} du$$

$$= \frac{4}{7} u^{7/4} - \frac{4}{5} u^{5/4} = \frac{4}{7} (x+1)^{7/4} - \frac{4}{5} (x+1)^{5/4}$$