1. Compute $\int e^{4x-9}\,dx$

$u = 4x - 9$

$du = 4\,dx$

$$\int e^u \frac{1}{4}\,du = \frac{1}{4}\int e^u du = \frac{1}{4}e^u = \frac{1}{4}e^{4x-9}$$

$$\int e^{4x-9}\,dx = \frac{1}{4}e^{4x-9}$$

2. Compute $\int x\sin(x^2+1)\,dx$

$u = x^2 + 1$

$du = 2x\,dx$ $\quad \frac{1}{2}du = x\,dx$ $\quad \frac{1}{2x}du = dx$

$$\int \sin(u)\frac{1}{2}\,du = \frac{1}{2}\cdot(-\cos(u))$$

$$= -\frac{1}{2}\cos(x^2+1)$$

$$\frac{d}{dx}\left(-\frac{1}{2}\cos(x^2+1)\right) = -\frac{1}{2}\cdot(-\sin(x^2+1))\cdot 2x$$

$$= \sin(x^2+1)\cdot x \;\checkmark$$

3. Compute $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$.

$u = \sqrt{x}$

$du = \frac{1}{2}x^{-1/2}\,dx$

$= \frac{1}{2}\frac{1}{\sqrt{x}}\,dx$

$2\,du = \frac{1}{\sqrt{x}}\,dx$

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = \int 2e^u\,du$$

$$= 2e^u$$

$$= 2e^{\sqrt{x}}$$

4. Compute $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$.

$$\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = 2e^{\sqrt{x}}\Big|_1^4 = 2e^{\sqrt{4}} - 2e^{\sqrt{1}}$$

$$= 2e^2 - 2e$$

$$= 2(e^2 - e)$$

5. Compute $\int \frac{\arctan(x)}{1+x^2}\,dx$

$u = \arctan(x)$

$du = \frac{1}{1+x^2}\,dx$

$= \int u\,du = \frac{1}{2}u^2$

$= \frac{1}{2}(\arctan(x))^2 + C$

6. Compute $\int \frac{x^3}{\sqrt{1-x^4}}\,dx$

$u = \sqrt{1-x^4}$

$u = 1 - x^4$

$du = -4x^3\,dx$

$du = \frac{1}{2}\frac{1}{\sqrt{1-x^4}} \cdot (-) \cdot 4x^3$

$= -2\frac{1}{\sqrt{1-x^4}}x^3\,dx$

$-\frac{1}{2}du = \frac{x^3}{\sqrt{1-x^4}}\,dx$

$\int -\frac{1}{2}\,du = -\frac{1}{2}\int 1\,du$

$= -\frac{1}{2}u$

$= -\frac{1}{2}\sqrt{1-x^4}$

7. Compute $\int \frac{x}{\sqrt{1-x^4}}\,dx$.

$u = x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$\frac{1}{2}\int \frac{1}{\sqrt{1-u^2}}\,du$

$= \frac{1}{2}\arcsin(u) = \frac{1}{2}\arcsin(x^2)$

8. Compute $\int \frac{\sec^2(x)}{\tan(x)}\,dx$

$u = \tan(x)$

$du = \sec^2(x)\,dx$

$= \int \frac{1}{u}\,du = \ln(|u|)$

$= \ln(|\tan(x)|) + C$

9. Compute $\int \sec^2(x)\tan(x)\,dx$

$u = \sec(x)$

$du = \sec(x)\tan(x)\,dx$

$\int u\,du = \frac{1}{2}u^2$

$= \frac{1}{2}\sec^2(x)$

$\int \sec^2(x)\tan(x)\,dx = \frac{1}{2}\sec^2(x) + C$

$u = \tan(x)$

$du = \sec^2(x)\,dx$

$\int u\,du = \frac{1}{2}u^2 = \frac{1}{2}\tan^2(x) + C$

$\int \sec^2(x)\tan(x)\,dx = \frac{1}{2}\tan^2(x) +$

10. Compute $\int \frac{\sin(\theta)}{1+\cos(\theta)}\,d\theta$

$u = 1 + \cos\theta$

$du = -\sin\theta\,d\theta$

$-du = \sin\theta\,d\theta$

$\int \frac{-du}{u} = -\ln(|u|)$

$= -\ln(|1+\cos\theta|)$

$= \ln\left(\frac{1}{1+\cos\theta}\right)$

$f(x)$

$\frac{d}{dx} f(x) = 0$ everywhere.

$\rightarrow f(x) = C$ for some C.

$\frac{d}{dx} f(x) = \frac{d}{dx} g(x)$ on an interval

$\frac{d}{dx}\left[f(x) - g(x)\right] = 0 \quad f(x) - g(x) = C$

$f(x) = g(x) + C$

$\sec^2(x) = \tan^2(x) + 1$

$\cos^2(x) + \sin^2(x) = 1 \Rightarrow 1 + \frac{\sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)}$

11. Compute $\int \frac{1}{x\ln(x)}\,dx$ $u = \ln(x)$ $du = \frac{1}{x}\,dx$

$$\int \frac{1}{u}\,du = \ln(|u|) = \ln(|\ln(x)|) + C$$

12. Compute $\int \frac{\sin(4/x)}{x^2}\,dx$ $u = \frac{4}{x}$ $du = \frac{-4}{x^2}$

$$\int -\frac{1}{4}\sin(u)\,du = \frac{1}{4}\cos(u) = \frac{1}{4}\cos\left(\frac{-4}{x^2}\right) + C$$

13. Compute $\int \frac{e^x}{e^x - 3}\,dx$ $u = e^x - 3$

$du = e^x\,dx$

$$\int \frac{du}{u} = \ln(|u|) = \ln(|e^x - 3|) + C$$