1. Gravel is being added to a pile at a rate of rate of $1+t^2$ tons per minute for $0 \le t \le 10$ minutes. If $G(t)$ is the amount of gravel (in tons) in the pile at time t, compute $G(10) - G(0)$. Then compute $G(10)$ assumeing $G(0) = 3$ tons.

change in amount of gravel between $t=0$ and $t=10$

$$G'(t) = 1+t^2$$

$$\int_0^{10} G'(t)\,dt = G(10) - G(0)$$

tons/minute

minutes

$$\int_0^{10} G'(t)\,dt = \int_0^{10} (1+t^2)\,dt$$

$$= t + \frac{t^3}{3}\Big|_0^{10}$$

$$= 10 + \frac{10^3}{3} = 343\tfrac{1}{3} \text{ tons}$$

$$G(10) - G(0) = 343\tfrac{1}{3} \text{ tons}$$

$$G(10) - 3 = 343\tfrac{1}{3} \text{ tons}$$

$$G(10) = 346\tfrac{1}{3} \text{ tons}$$

2. There is a population of bacteria with 3000 cells at time $t = 0$. It grows at a rate of $1000 \cdot 5^t$ cells per hour. What is the population of the bacteria at time $t = 1$ hours?

$P(t)$: # of cells at time t hours

$P(0) = 3000$

$P'(t) = 1000 \cdot 5^t$

$$\frac{d}{dt} e^t = e^t$$

$$\frac{d}{dt} e^{7t} = 7e^{7t}$$

$$\frac{d}{dt} e^{at} = ae^{at}$$

$$\boxed{\frac{d}{dt} \frac{1}{a} e^{at} = e^{at}}$$

$$\frac{d}{dt} \frac{1}{7} e^{7t} = e^{7t}$$

$$\frac{d}{dt} 5^t = \ln(5) 5^t$$

$$e^{at}$$

$$P(1) - P(0) = \int_0^1 P'(t)\, dt$$

$$= \int_0^1 1000 \cdot 5^t\, dt$$

$$= 1000 \int_0^1 5^t\, dt$$

$$= 1000 \int_0^1 e^{\ln(5) t}\, dt$$

$$= 1000 \cdot \frac{1}{\ln(5)} e^{\ln(5) t} \Big|_0^1$$

$$= \frac{1000}{\ln(5)} \left[e^{\ln(5) \cdot 1} - e^{\ln(5) \cdot 0} \right]$$

$$= \frac{1000}{\ln(5)}\left[\,5 - 1\,\right]$$

$$= \frac{4000}{\ln(5)} = 2485 \text{ cells}$$

$$P(1) - P(0) = 2485 \text{ cells}$$

$$P(1) - 3000 = 2485$$

$$P(1) = 5485 \text{ cells}$$

5^t

$$5 = e^{\ln(5)}$$

$$(a^b)^c = a^{bc}$$

$$5^t = \left(e^{\ln(5)}\right)^t$$

$$= e^{\ln(5)t}$$

3. Water flows from a tank at a rate of $r(t) = 3t^2 - t^3$ liters per minute from $t = 0$ to $t = 3$ minutes.

a. Compute $r(0)$, $r(1)$ and $r(3)$, and explain what these quantities mean in everyday language. Your answer should include units.

$$\left.\begin{aligned} r(0) &= 3\cdot 0^2 - 0^3 = 0 \\ r(1) &= 3\cdot 1^2 - 1^3 = 2 \\ r(3) &= 3\cdot 3^2 - 3^3 = 0 \end{aligned}\right] \ell/\text{min}$$

The flow rate was 0 ℓ/min at $t=0$ and 3, but 2 ℓ/min at $t=1$.

b. Compute the total amount of water that drains from the tank from time $t = 0$ to $t = 3$.

$V(t) \to$ volume of water that escaped the tank

$V(t) \to \ell$

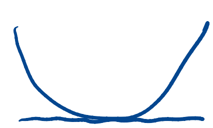
$$V'(t) = r(t)$$

$$V(3) - V(0) = \int_0^3 V'(t)\,dt$$
$$= \int_0^3 3t^2 - t^3\,dt$$

c. At what time is the rate of flow at a maximum? (Only consider t in the interval $[0, 3]$.)

$$r(t) = 3t^2 - t^3$$

$$r'(t) = 6t - 3t^2$$



$$= 3t(2-t)$$

$r = 0, 2, 3$

$r'(t) = 0$ at $t=0$, $t=2$

$r(0) = 0$, $r(3) = 0$

$r(2) = 3\cdot 2^2 - 2^3$

$= 12 - 8 = 4$

$$V(3) - V(0) = \int_0^3 V'(t)\,dt$$

$$= \int_0^3 3t^2 - t^3\,dt$$

$$= t^3 - \frac{t^4}{4}\Big|_0^3$$

$$= 3^3 - \frac{3^4}{4} = 6.75\ \ell$$

4. Challenge! Compute

$$\frac{d}{dx}\int_5^{x^3} \cos(\sqrt{s})\, ds.$$

Hint: Let $H(x) = \int_5^x \cos(\sqrt{s})\, ds$. You're interested in $H(x^3)$. Apply the Chain Rule!

$$F(x) = \int_5^x \cos(\sqrt{s})\, ds$$

$$F'(x) = \cos(\sqrt{x}) \quad \text{(FTC Part I!)}$$

$$\frac{d}{dx} F(x^3) = F'(x^3) \cdot 3x^2 \quad \text{(chain rule)}$$

$$\frac{d}{dx}\int_5^{x^3} \cos(\sqrt{s})\, ds = \cos\left(\sqrt{x^3}\right) \cdot 3x^2$$