1. Compute $\int_1^2 \frac{t^3 - 3t^2}{t^4}\, dt$.

$$\int_1^2 \frac{t^3-3t^2}{t^4}\,dt = \int_1^2 \frac{1}{t} - \frac{3}{t^2}\,dt = \ln(|t|) + \frac{3}{t}\Big|_1^2$$

$$= \ln(2) - \ln(1) + \frac{3}{2} - \frac{3}{1}$$

$$= \ln(2) - 3/2$$

2. Compute $\frac{d}{dx}\int_5^x \cos(\sqrt{s})\, ds$.

$\cos(\sqrt{x})$ (!)

3. Compute $\int x^2(3-x)\, dx$

$$\int 3x^2 - x^3\,dx = x^3 - \frac{x^4}{4} + C$$

4. Compute $\int 9\sqrt{x} - 3\sec(x)\tan(x)\, dx$

$$\int 9\sqrt{x} - 3\sec(x)\tan(x)\,dx = 6x^{3/2} - 3\sec(x) + C$$

5. Snow is falling on my garden at a rate of

$[t] = \text{hours}$

$$A(t) = 10e^{-2t}$$

$[A(t)] = \text{kg}/\text{hour}$

kilograms per hour for $0 \le t \le 2$, where t is measured in hours.

$[m(t)] = \text{kg}$

a. If $m(t)$ is the total mass of snow on my garden, how are $m(t)$ and $A(t)$ related to each other?

$$A(t) = m'(t)$$

b. What does $m(2) - m(0)$ represent?

Change in mass of snow on garden between $t=0$ and $t=2$ hours.

c. Find an antiderivative of $A(t)$.

$$-5e^{-2t}$$

d. Compute the total amount of snow accumulation from $t = 0$ to $t = 1$.

$$\int_0^1 10e^{-2t}\,dt = -5e^{-2t}\Big|_0^1 = -5e^{-2\cdot 1} - \left(-5e^{-2\cdot 0}\right)$$

$$= -5e^{-2} + 5$$

$$= 5(1-e^{-2})$$

$$= 4.32 \text{ kg}$$

e. Compute the total amount of snow accumulation from $t = 0$ to $t = 2$.

$$\int_0^2 10e^{-2t}\,dt = -5e^{-2t}\Big|_0^2 = 5(1-e^{-4})$$

f. From the information given so far, can you compute $m(2)$?

No

g. Suppose $m(0) = 9$. Compute $m(1)$ and $m(2)$.

$$m(1) = m(0) + (m(1) - m(0)) = 9 + 5(1-e^{-2})$$

$$m(2) = m(0) + (m(2) - m(0)) = 9 + 5(1-e^{-4})$$

6. A airplane is descending. Its rate of change of height is $r(t) = -4t + \frac{t^2}{10}$ meters per second.

a. if $A(t)$ is the altitude of the airplane in meters, how are $A(t)$ and $r(t)$ related?

$$A'(t) = r(t)$$

$$\sum_{k=1}^{n} r(t_k^*) \cdot \Delta t$$

b. What physical quantity does $\int_1^3 r(t)\,dt$ represent?

$[r(t)] = m/s, \quad [dt] = s$

Units: m

$\boxed{\text{Change in height (in m) of the airplane from time } t=1 \text{ to } t=3 \text{ seconds}}$

c. Compute $A(3) - A(1)$.

Change in height from $t=1$ to $t=3$ seconds

$$\int_1^3 r(t)\,dt = \int_1^3 -4t + \frac{t^2}{10}\,dt = -2t^2 + \frac{t^3}{30}\Bigg|_1^3$$

$$= \left[-2\cdot 3^2 + \frac{3^3}{30}\right] - \left[-2\cdot 1^2 + \frac{1^3}{30}\right]$$

-3.8 m

7. Gravel is being added to a pile at a rate of rate of $1 + t^2$ tons per minute for $0 \le t \le 10$ minutes. If $G(t)$ is the amount of gravel (in tons) in the pile at time t, compute $G(10) - G(0)$.