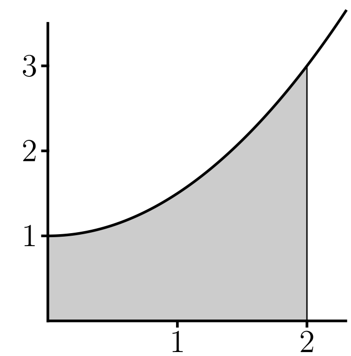
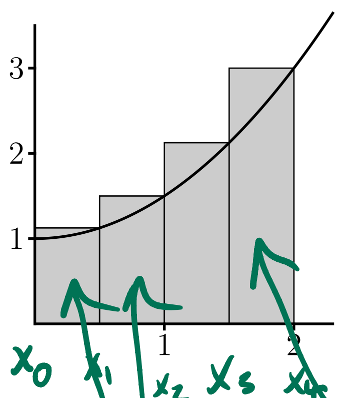
The goal of the first part is to estimate the area under the curve $y = \frac{1}{2}x^2 + 1$ and above the x-axis on the interval $[0, 2]$.



1. Use $n = 4$ rectangles with right-hand endpoints. Overestimate or underestimate?

$f(x) = \frac{1}{2}x^2 + 1$

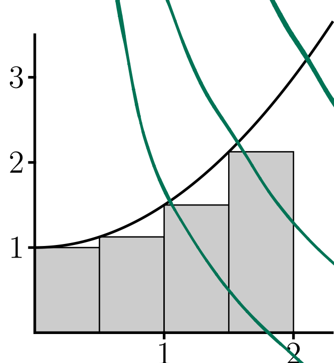


$\Delta x = \frac{1}{2}$

$x_0 : 0$
$x_1 : 0.5$
$x_2 : 1$
$x_3 : 1.5$
$x_4 : 2$

$f(x_1) : 1 + \frac{1}{2}\left(\frac{1}{2}\right)^2 = 1\,1/8$
$f(x_2) : 1.5$
$f(x_3) : \frac{1}{2}(1.5)^2 + 1 = 2.125$
$f(x_4) : 3$

2. Use $n = 4$ rectangles with left-hand endpoints. Overestimate or underestimate?



$\frac{1}{2} \cdot 3 = \Delta x f(x_4)$

$\frac{1}{2} \cdot (2.125) = \Delta x f(x_3)$

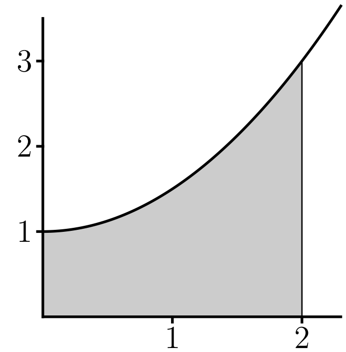
$\frac{1}{2}\, 1.5 = \Delta x f(x_2)$

$\frac{1}{2} \cdot 1\,1/8 = \Delta x f(x_1)$

3. From your last two answers, give your best estimate for the area.

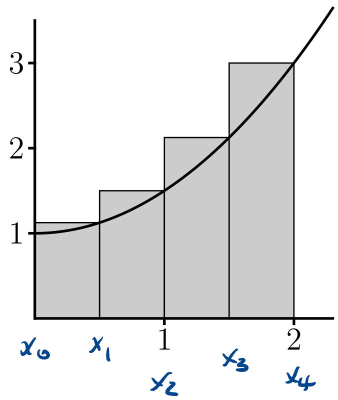
$\frac{1}{2} \cdot 3 + \frac{1}{2} \cdot 2.125 + \frac{1}{2} \cdot 1.5 + \frac{1}{2} \cdot 1\,1/8 = 3.875$

The goal of the first part is to estimate the area under the curve $y = \frac{1}{2}x^2 + 1$ and above the x-axis on the interval $[0, 2]$.



1. Use $n = 4$ rectangles with right-hand endpoints. Overestimate or underestimate?

$f(x) = \frac{1}{2}x^2 + 1$



$\Delta x = \frac{1}{2}$

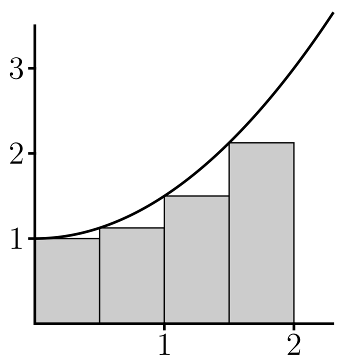
$x_0 = 0$
$x_1 = 1/2$
$x_2 = 1$
$x_3 = 3/2$
$x_4 = 2$

$A = \Delta x f(x_1) + \Delta x f(x_2) + \Delta x f(x_3) + \Delta x f(x_4)$

$f(x_1) = 1 + \frac{1}{2}(1/2)^2$
$f(x_2) = 1 + \frac{1}{2}(1)^2$
$f(x_3) = 1 + \frac{1}{2}(3/2)^2$
$f(x_4) = 1 + \frac{1}{2}(2)^2$

$= 3.875$

2. Use $n = 4$ rectangles with left-hand endpoints. Overestimate or underestimate?



$\Delta x = \frac{1}{2}$

$x_0 = 0$
$x_1 = 1/2$
$x_2 = 1$
$x_3 = 3/2$
$x_4 = 2$

$A = \Delta x f(x_0) + \Delta x f(x_1) + \Delta x f(x_2) + \Delta x f(x_3)$

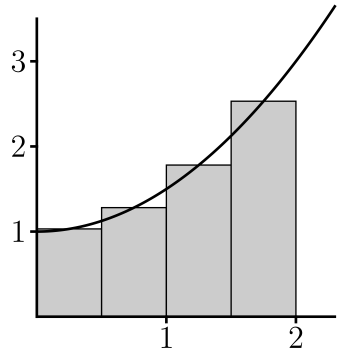
$f(x_0) = 1 + \frac{1}{2}0^2$
$f(x_1) = 1 + \frac{1}{2}(1/2)^2$
$f(x_2) = 1 + \frac{1}{2}(1)^2$
$f(x_3) = 1 + \frac{1}{2}(3/2)^2$

$= 2.875$

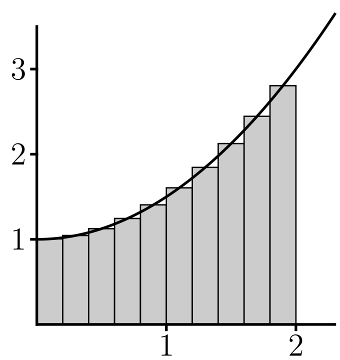
3. From your last two answers, give your best estimate for the area.

$\frac{1}{2}(3.375 + 2.875) = 3.375$

4. Use $n = 4$ rectangles with midpoints. Overestimate or underestimate?



5. Use $n = 10$ rectangles with midpoints.



6. Suppose the odometer on our car is broken and we want to estimate the distance driven over a 1.5 hour time period. We take speedometer readings every 15 minutes and then record them in the table below. Estimate the distance traveled by the car. What method are you using?

	t_0	t_1	t_2	t_3	t_4	t_5	t_6
Time (minutes)	0	15	30	45	60	75	90
Velocity (mi/h)	17	21	24	29	32	31	28
	v_0	v_1	v_2	v_3	v_4	v_5	v_6

$\Delta t = \frac{1}{4}$ hour

$\Delta x_1 = v_0 \Delta t = 17 \cdot \frac{1}{4}$

$\Delta x_2 = v_1 \Delta t = 21 \cdot \frac{1}{4}$

$\Delta x_3 = v_2 \Delta t = 24 \cdot 1/4$

$\Delta x_4 = v_3 \Delta t = 29 \cdot 1/4$

$\Delta x_5 = v_4 \Delta t = 31 \cdot 1/4$

$\Delta x_6 = v_5 \Delta t = 28 \cdot 1/4$

Distance:

$17 \cdot \frac{1}{4} + 21 \cdot \frac{1}{4} + 24 \cdot \frac{1}{4} + 29 \cdot \frac{1}{4} + 31 \cdot \frac{1}{4} + 28 \cdot \frac{1}{4}$

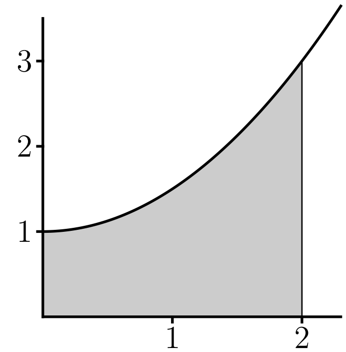
$= (17 + 21 + 24 + 29 + 31 + 28)\frac{1}{4}$

$= 41.25$ miles

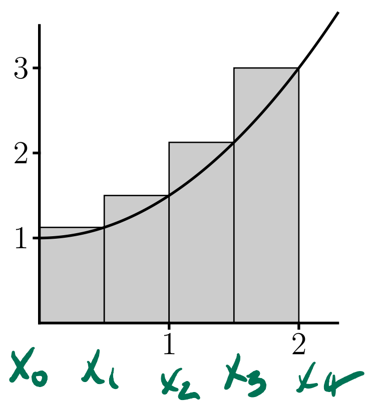
7. Oil leaked out of a tank at a rate of $r(t)$ liters per hour. The rate decreased as time passed and values of the rate at 2 hour time intervals are shown in the table. Estimate how much oil leaked out. What method are you using? Is it an overestimate or an underestimate.

t (h)	0	2	4	6	8	10
r(t) (L/h)	8.7	7.6	6.8	6.2	5.7	5.3

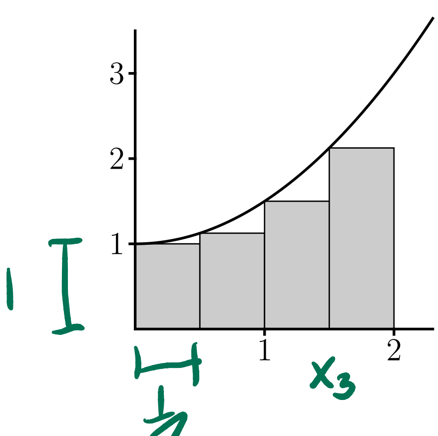
The goal of the first part is to estimate the area under the curve $y = \frac{1}{2}x^2 + 1$ and above the x-axis on the interval $[0, 2]$.



1. Use $n = 4$ rectangles with right-hand endpoints. Overestimate or underestimate?



2. Use $n = 4$ rectangles with left-hand endpoints. Overestimate or underestimate?



$\Delta x = \frac{1}{2}$ $x_0 = 0$ $f(x_0) = \frac{1}{2}0^2+1 = 1$

$x_1 = 1/2$ $f(x_1)$

$x_2 = 1$ $f(x_2)$

$x_3 = 1.5$ $f(x_3)$

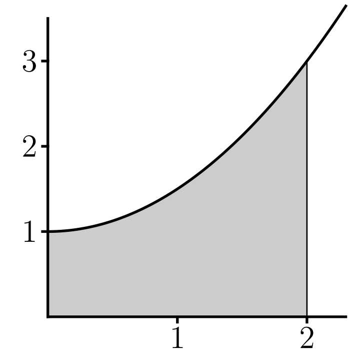
$x_4 = 2$

3. From your last two answers, give your best estimate for the area.

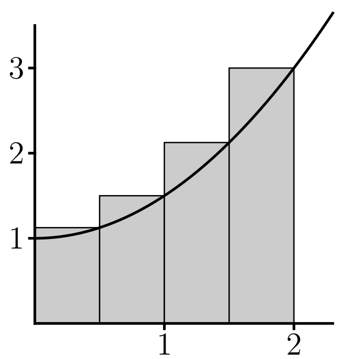
$\underbrace{f(x_0)\Delta x}+f(x_1)\Delta x+f(x_2)\Delta x+f(x_3)\Delta x$

$\left[1\cdot\frac{1}{2} + 1.5\,\frac{1}{2} + 0 \cdot 0\right] = 2.875$

The goal of the first part is to estimate the area under the curve $y = \frac{1}{2}x^2 + 1$ and above the x-axis on the interval $[0, 2]$.



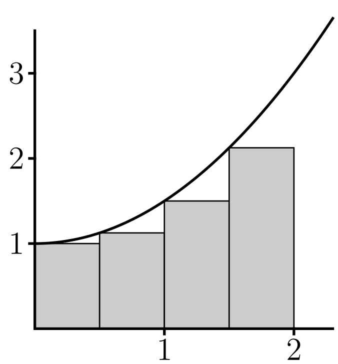
1. Use $n = 4$ rectangles with right-hand endpoints. Overestimate or underestimate?



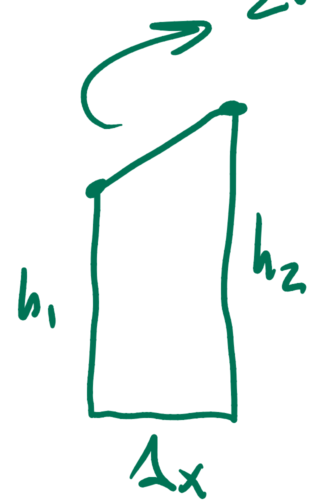
$A \approx 3.875$

2. Use $n = 4$ rectangles with left-hand endpoints. Overestimate or underestimate?

$\frac{1}{2}(h_1+h_2)\Delta x$



$A \approx 2.875$



3. From your last two answers, give your best estimate for the area.

$$A \approx \frac{1}{2}\left[3.875 + 2.875\right] = 3.375$$

$\Delta x = \frac{1}{2}$

$x_0 : 0$
$x_1 : 0.5$
$x_2 : 1$
$x_3 : 1.5$
$x_4 : 2$

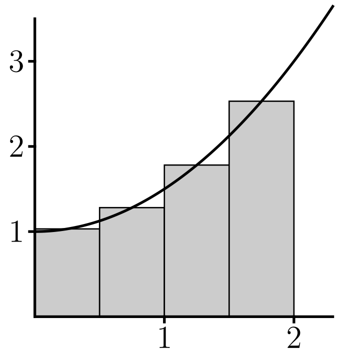
$f(x_1) : 1 + \frac{1}{2}\left(\frac{1}{2}\right)^2 = 1\,{}^1\!/_8$
$f(x_2) : 1.5$
$f(x_3) : \frac{1}{2}(1.5)^2 + 1 = 2.125$
$f(x_4) : 3$

$$f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\Delta x$$

$$\underbrace{\big(f(x_1) + f(x_2) + f(x_3) + f(x_4)\big) \cdot \Delta x}$$

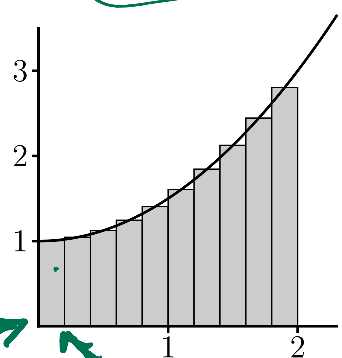
$$\rightarrow 3.875$$

4. Use n = 4 rectangles with midpoints. Overestimate or underestimate?



right hand points

5. Use n = 10 rectangles with ~~midpoints~~.



x_0

x_1

x_{10}

$\Delta x = \frac{2}{10} = \frac{1}{5} = 0.2$

$x_0 = 0$

$x_1 = 1/5$

$x_2 = 2/5$

$x_3 = 3/5$

$\vdots$

$x_{10} = 10/5 = 2$

$f(x_1) = \frac{1}{2}\left(\frac{1}{5}\right)^2 + 1$

$f(x_2) = \frac{1}{2}\left(\frac{2}{5}\right)^2 + 1$

$\vdots$

$f(x_{10}) = 3$

$f(x_1)\Delta x + f(x_2)\Delta x + \cdots + f(x_{10})\Delta x$

$[f(x_1) + f(x_2) + \cdots + f(x_{10})]\Delta x = 3.54$

6. Suppose the odometer on our car is broken and we want to estimate the distance driven over a 1.5 hour time period. We take speedometer readings every 15 minutes and then record them in the table below. Estimate the distance traveled by the car. What method are you using?

Time (minutes)	0	15	30	45	60	75	90
Velocity (mi/h)	17	21	24	29	32	31	28

First time interval: 17 mph for 15 min $= \frac{1}{4}$ hr

$17 \cdot \frac{1}{4} = 4\frac{1}{4}$ miles.

2nd time interval: $21 \cdot \frac{1}{4}$ miles

Our first 30 mins $(17+21) \cdot \frac{1}{4}$

38.5 miles

Over 90 mins: $17 \cdot \frac{1}{4} + 21 \cdot \frac{1}{4} + 24 \frac{1}{4} + 29 \cdot \frac{1}{4} + 32 \cdot \frac{1}{4} + 31 \cdot \frac{1}{4}$

7. Oil leaked out of a tank at a rate of $r(t)$ liters per hour. The rate decreased as time passed and values of the rate at 2 hour time intervals are shown in the table. Estimate how much oil leaked out. What method are you using? Is it an overestimate or an underestimate.

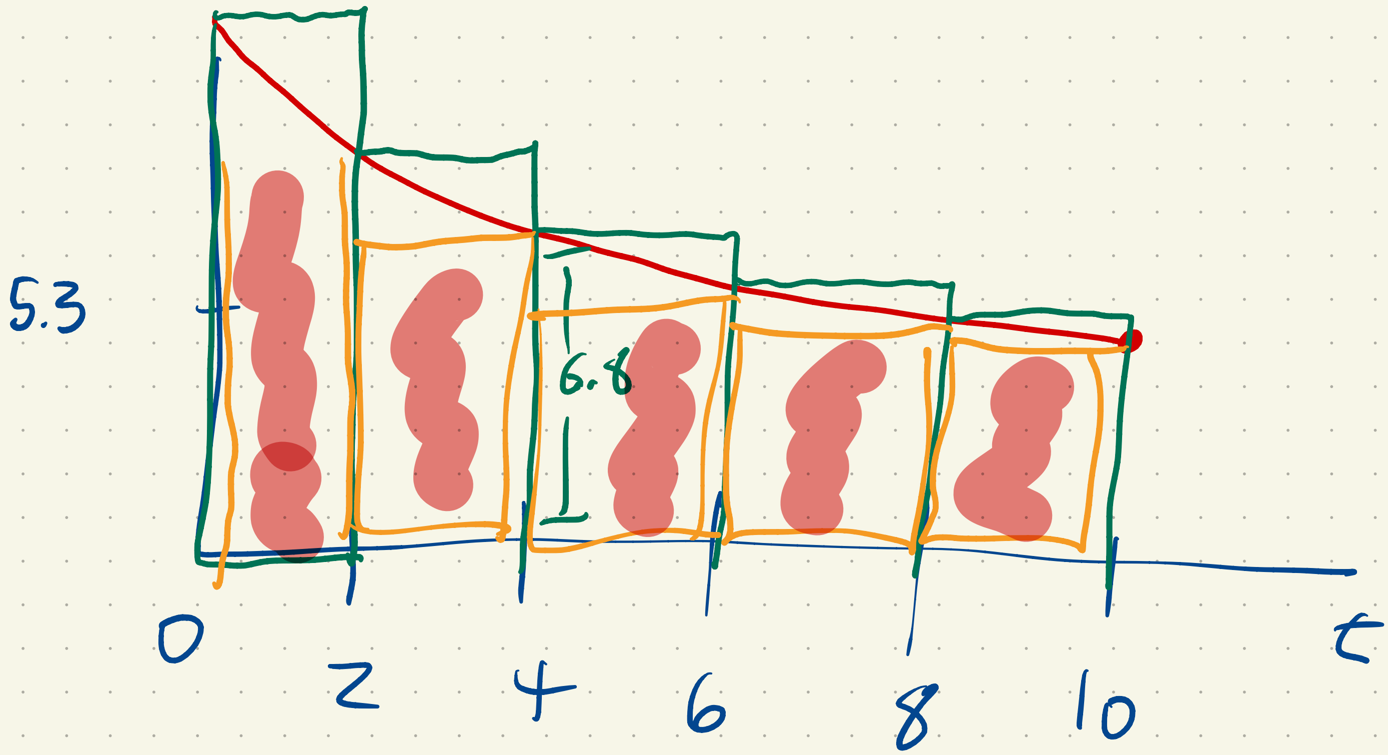
t (h)	0	2	4	6	8	10
r(t) (L/h)	8.7	7.6	6.8	6.2	5.7	5.3

First two hours: $\approx 8.7 \cdot 2$ ℓ leaked.

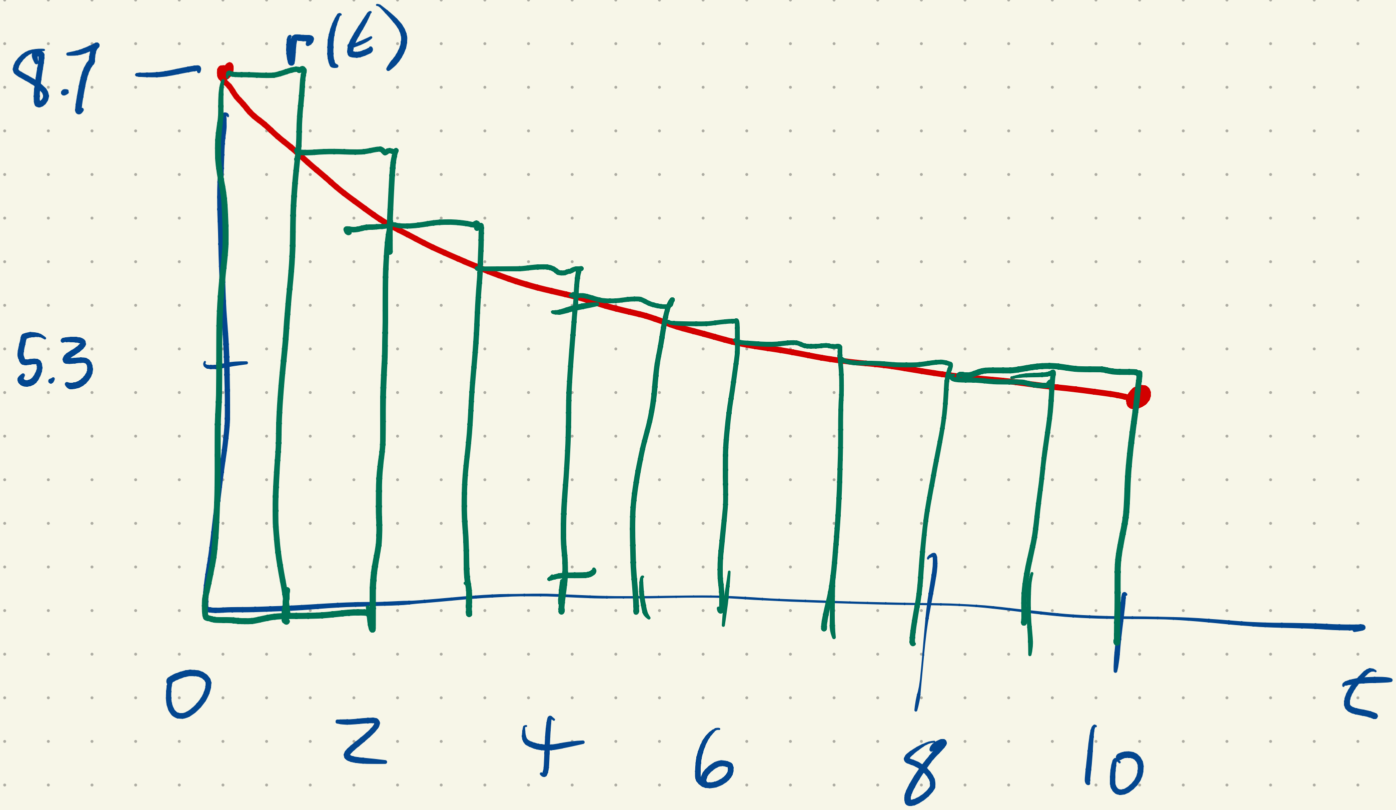
Next two hours: $\approx 7.6 \cdot 2$ ℓ

Estimate: $(8.7 + 7.6 + 6.8 + 6.2 + 5.7) \cdot 2 = 70$ ℓ

Estimate: $(7.6 + 6.8 + 6.2 + 5.7 + 5.3) \cdot 2 = 63.2$ ℓ

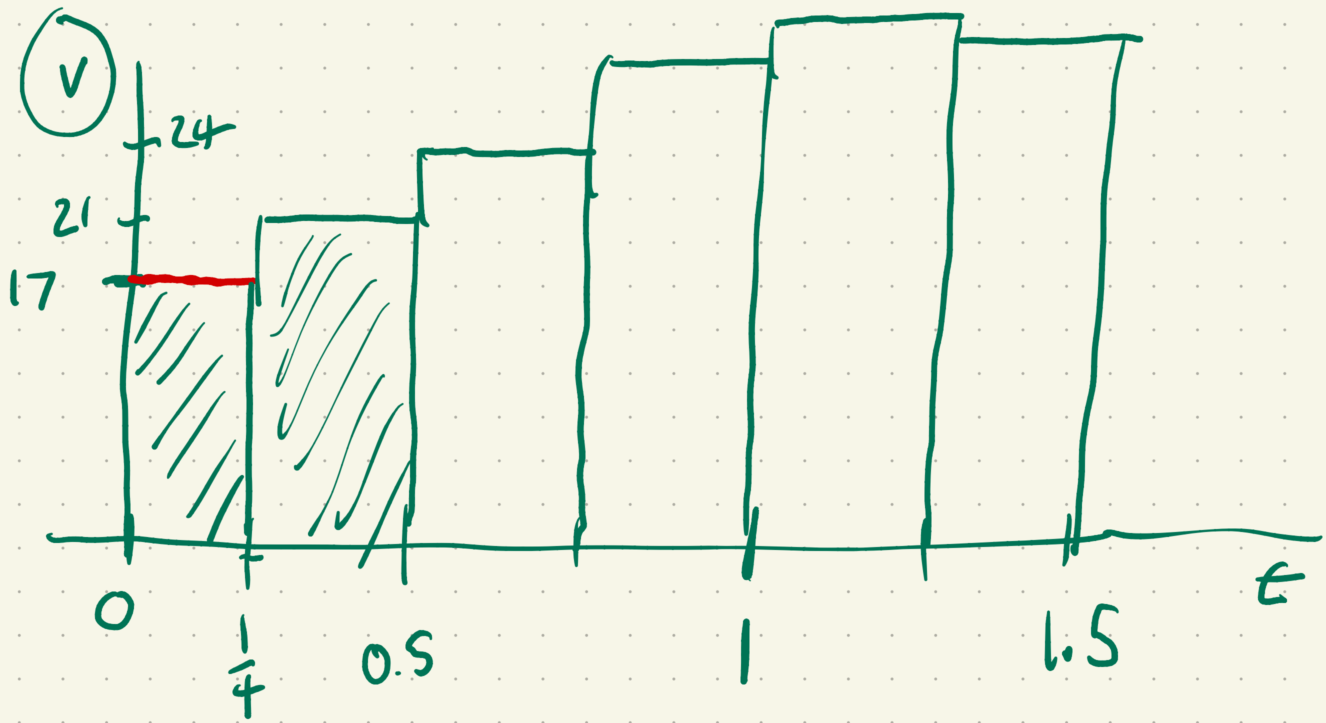


5.3
6.8
0
2
4
6
8
10
t



8.7
r(t)
5.3
0
2
4
6
8
10
t

$$17\cdot\frac{1}{4} + 21\cdot\frac{1}{4} + 24\frac{1}{4} + 29\cdot\frac{1}{4} + 32\cdot\frac{1}{4} + 31\cdot\frac{1}{4}$$



$$\frac{1}{4}\cdot 17 + \frac{1}{4}\cdot 21 + \frac{1}{4}\cdot 24$$