An **antiderivative** of a function $f(x)$ is a function $F(x)$ with $F'(x) = f(x)$.

If $F(x)$ is a particular antiderivative of $f(x)$, then so is $F(x) + C$ for any constant C.

If the domain of $f(x)$ is an interval, and if $F(x)$ is a particular rantiderivative of $f(x)$, then any antiderivative has the form $F(x) + C$ for some constant C.

If $F(x)$ and $G(x)$ are antiderivatives of $f(x)$ and $g(x)$ then

- $aF(x)$ is an antiderivative of $af(x)$ for any constant a.
- $F(x) + G(x)$ is an antiderivative of $f(x) + g(x)$.

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1. Find a particular antiderivative of $x - x^2 + 9$.

$$\frac{1}{2}x^2 - \frac{x^3}{3} + 9x$$

2. Find all antiderivatives of $x - x^2 + 9$.

$$\frac{1}{2}x^2 - \frac{x^3}{3} + 9x + C$$

3. Find an antiderivative of $1/x^2$.

$$F(x) = \frac{-1}{x} \quad \text{since} \quad F'(x) = \frac{-1\cdot(-1)}{x^2} = \frac{1}{x^2}$$

4. If $F(x)$ is your answer to the previous problem, does every antiderivative of $1/x^2$ have the form $F(x) + C$ for some constant C?

No.

$$F(x) = \begin{cases} -\frac{1}{x} & x > 0 \\ -\frac{1}{x} + 9 & x < 9 \end{cases}$$

works

5. For each of the following functions, find a particular antiderivative.

Function	Antiderivative
x	$\frac{1}{2}x^2$
x^2	$\frac{1}{3}x^3$
x^3	$\frac{1}{4}x^4$
x^k $(k \neq -1)$	$\frac{1}{k+1}x^{k+1}$
x^{-1} for $x > 0$	$\ln(x)$
x^{-1} for $x < 0$	$\ln(-x)$
x^{-1} for all x	$\ln(\lvert x\rvert)$

$\hookrightarrow x \neq 0$

Function	Antiderivative
$\sin(x)$	$-\cos(x)$
$\cos(x)$	$\sin(x)$
e^x	e^x
$1/(1+x^2)$	$\arctan(x)$
$\sec^2(x)$	$\tan(x)$
$\sec(x)\tan(x)$	$\sec(x)$
1	x

6. Compute three different antiderivatives of $f(x) = x^{20} + 4x^{10} + 8$

$$F(x) = \frac{x^{21}}{21} + \frac{4x^{11}}{11} + 8x + \begin{cases} 19 \\ \sqrt{\pi} \\ e^2 \end{cases}$$

7. Compute an antiderivative of $f(t) = \dfrac{5\sec t \tan t}{3} - 4\sin t - \dfrac{1}{t} + e^2$

$$\frac{5}{3}\sec(t) + 4\cos(t) - \ln(|t|) + e^2 t$$

8. Compute an antiderivative of $f(x) = \cos(3x)$.

$$\frac{1}{3}\sin(3x)$$

9. Compute the antiderivative of $f(t) = t^2$ that equals 5 when $t = 2$.

$$F(t) = \frac{t^3}{3} + C$$

$$F(2) = \frac{8}{3} + C = 5$$

$$C = 7/3$$

$$F(t) = \frac{t^3}{3} + \frac{7}{3}$$

10. A particle moves in a straight line and has acceleration given by $a(t) = 5\cos t - 2\sin t$. Its initial velocity is $v(0) = -6$ m/s and its initial position is $s(0) = 2$ m. Find its position function $s(t)$.

$$s''(t) = 5\cos(t) - 2\sin(t)$$

$$s'(t) = 5\sin(t) + 2\cos(t) + C_1$$

$$s(t) = -5\cos(t) + 2\sin(t) + C_1 t + C_0$$

$$s(0) = -5 + C_0 = 2 \Rightarrow C_0 = 7$$

$$s'(0) = 2 + C_1 = -6 \Rightarrow C_1 = -8$$

$$s(t) = -5\cos(t) + 2\sin(t) - 8t + 7$$

11. A stone is dropped from a cliff and hits the ground three seconds later. How high is the cliff? (Acceleration due to gravity is $9.8\,\mathrm{m/s^2}$.)

$$h''(t) = -9.8$$

$$h'(t) = -9.8t + C_1$$

$$h(t) = -9.8\frac{t^2}{2} + C_1 t + C_0$$

$$h'(0) = 0 \Rightarrow C_1 = 0$$

$$h(3) = 0 \Rightarrow -\frac{9.8 \cdot 9}{2} + C_0 = 0 \Rightarrow C_0 = +44.1$$

$$h(0) = -9.8 \cdot 0^2/2 + 44.1$$ Cliff height: 44.1 m

12. What constant acceleration is needed to take a car from 10 mph to 60 mph in 5 seconds?

$x''(t) = a$, const

$$\underbrace{x'(t)}_{\text{velocity, } v(t)} = at + C$$

$v(0) = 10 \Rightarrow C = 10$

$$v(t) = at + 10$$

$$v(5) = a \cdot 5 + 10 = 60$$

$$\Rightarrow a = 10 \text{ mph/s}$$

$$\approx 4.4 \text{ m/s}^2$$