1. Follow the guidelines from the previous worksheet to sketch the graph of

$$f(x) = \frac{2}{x} + \ln(x).$$

$$\underbrace{\frac{d}{dx}\ln(x)=\frac{1}{x}}_{\to\, x>0}$$

a. What is the function's domain?

$(0, \infty)$

b. Does this function have any symmetry?

$f(-1) = -f(1)$

$f(-x) = -f(x)$

not allowed

No symmetry

c. Find a few choice values of x to evaluate the function at.

$$f(1) = \frac{2}{1} + \ln(1) = 2$$

$$\lim_{x\to 0^+} x\ln x$$

d. What behaviour occurs for this function at $\pm\infty$?

$$\lim_{x\to\infty} \frac{2}{x} + \ln(x) = 0 + \infty = \infty$$

e. Does the function have any vertical asymptotes? Where?

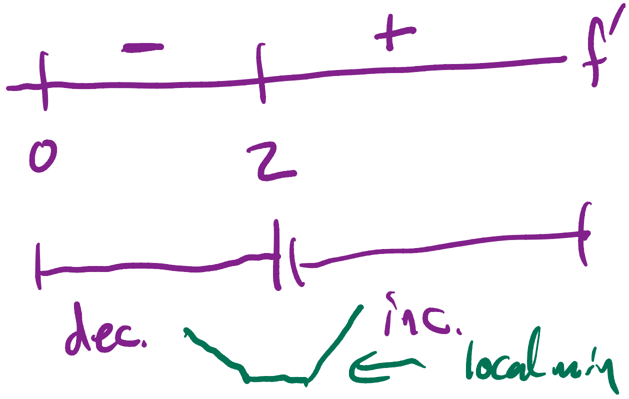
$$\lim_{x\to 0^+} \frac{2}{x} + \ln(x) = \lim_{x\to 0^+} \frac{1}{x}\left[2 + x\ln(x)\right] = \infty\left[2+0\right] = \infty$$

f. Find intervals where f is increasing/decreasing and identify critical points.

$f'(x) > 0$ $\quad$ $f'(x) < 0$ $\quad$ $\to f'(x) = 0$ or DNE

$$f'(x) = \frac{d}{dx}\left[\frac{2}{x} + \ln(x)\right]$$

$$= -\frac{2}{x^2} + \frac{1}{x} = \frac{x-2}{x^2}$$



$$\lim_{x\to0^+} x\ln(x) = \lim_{x\to0^+} \frac{\ln(x)}{1/x} \overset{\frac{\infty}{\infty}}{=} \lim_{x\to0^+} \frac{\frac{1}{x}}{-1/x^2}$$

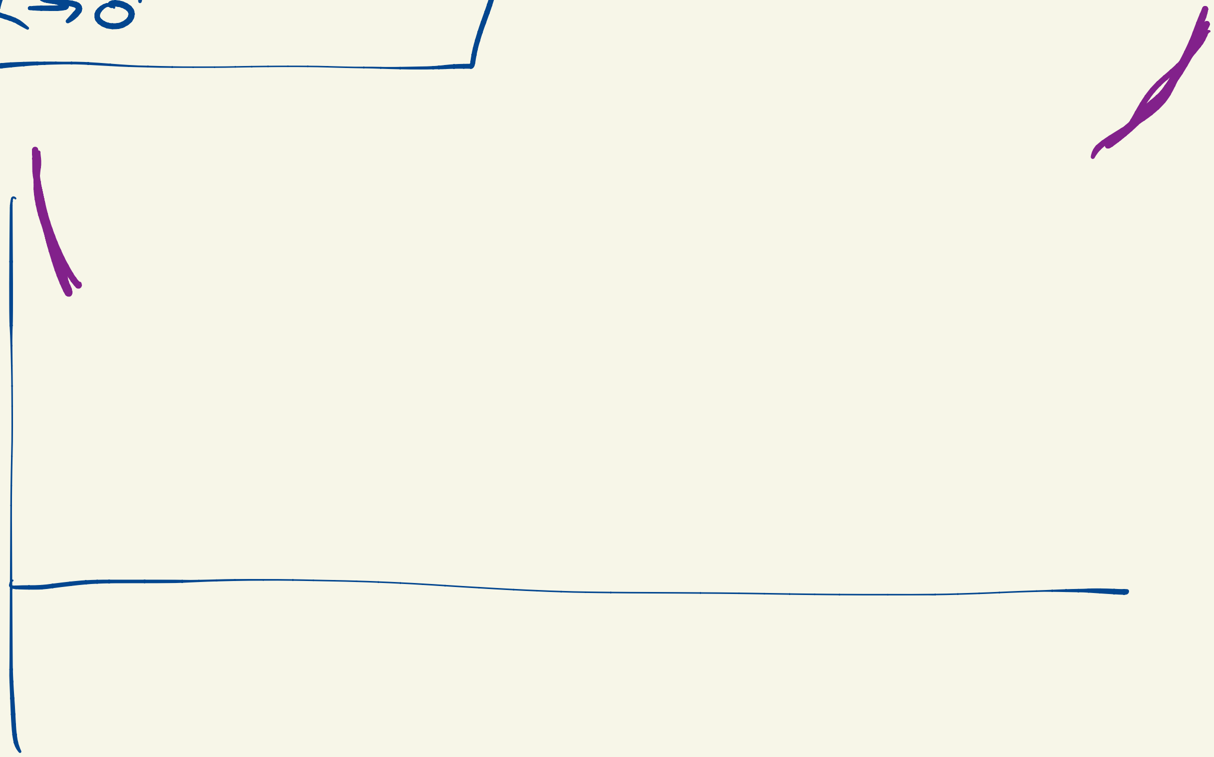
$$\uparrow \quad \uparrow$$

$$0 \cdot -\infty \qquad \frac{-\infty}{\infty}$$

$$= \lim_{x\to0^+} -x$$

$$= 0$$

$$\boxed{\lim_{x\to0^+} x\ln(x) = 0}$$



increasing: $(2,\infty)$

decreasing: $(0,2)$

critical point, $x=2$, local min

g. Classify each critical point as a local min/max/neither.

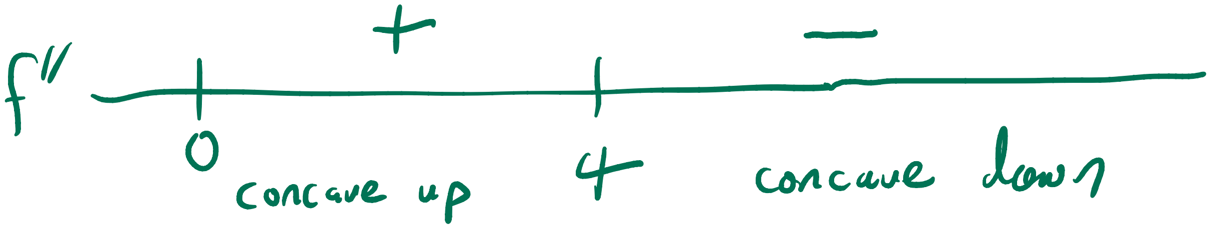
$x = 2$, local min.

h. Find intervals where f is concave up/concave down and identify points of inflection

$$f'(x) = \frac{x-2}{x^2}$$

$$f''(x) = \frac{1\cdot x^2 - (x-2)\cdot 2x}{(x^2)^2} = \frac{x^2 - 2x^2 + 4x}{x^4}$$

$$= \frac{x - 2x + 4}{x^3} = \frac{4-x}{x^3}$$

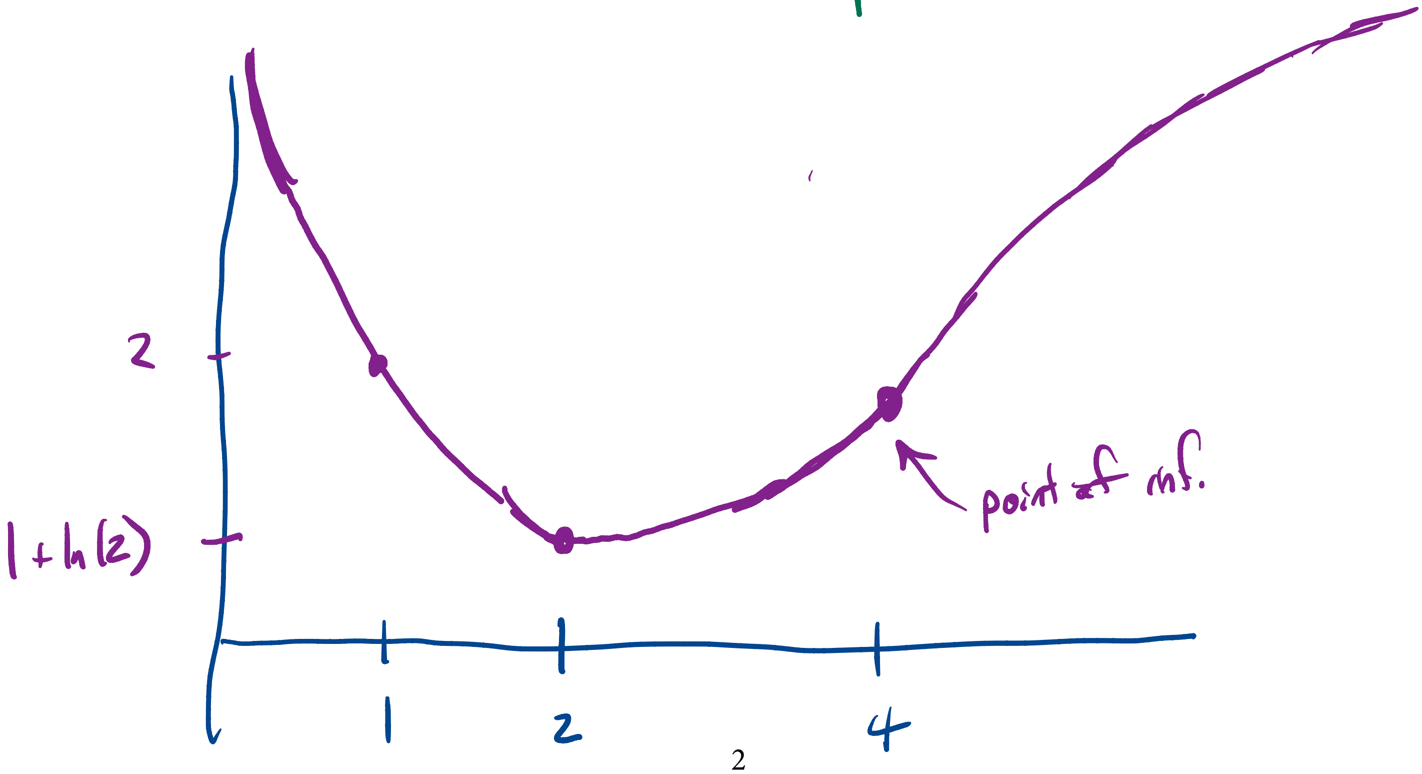


concave up

concave down

point of inflection

i. Sketch the graph of the function



2. Follow the guidelines from the previous worksheet to sketch the graph of

$$f(x) = x\sqrt{4-x^2}.$$

a. What is the function's domain?

$f(-x) = f(x)$

$f(-x) = -f(x)$

$[-2, 2]$

b. Does this function have any symmetry?

$f(-x) = (-x)\sqrt{4-(-x)^2} = -x\sqrt{4-x^2} = -f(x)$

odd

c. Find a few choice values of x to evaluate the function at.

$x = 0, \pm 2$ $f(0) = 0$, $f(2) = 0$, $f(-2) = 0$

d. What behaviour occurs for this function at ±∞?

NA

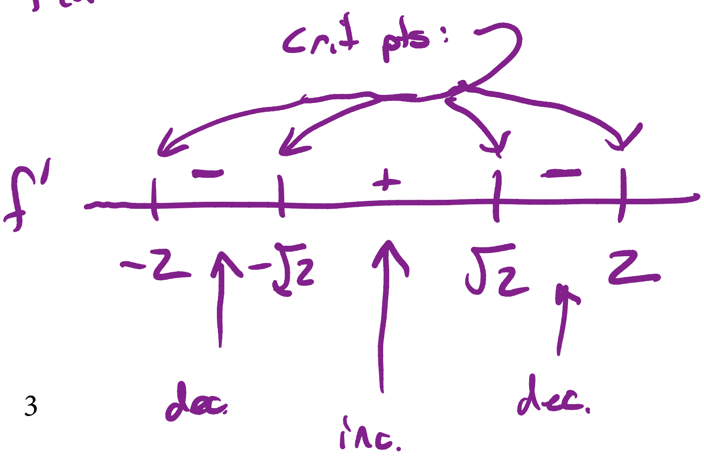
e. Does the function have any vertical asymptotes? Where?

No asymptotes.

f. Find intervals where f is increasing/decreasing and identify critical points.

$f'(x) > 0$ $f'(x) < 0$ ↳ $f'(x) = 0$, DNE

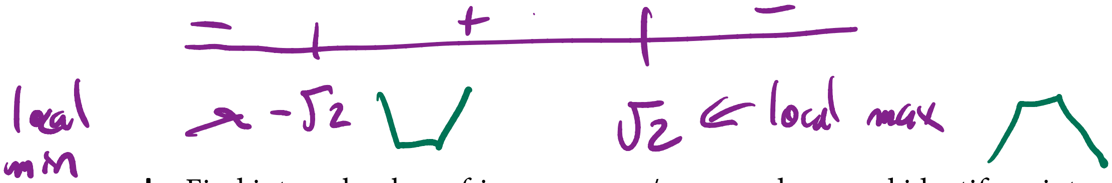
$$f'(x) = \frac{2(2-x^2)}{\sqrt{4-x^2}}$$



$f'(2), f'(-2)$: DNE

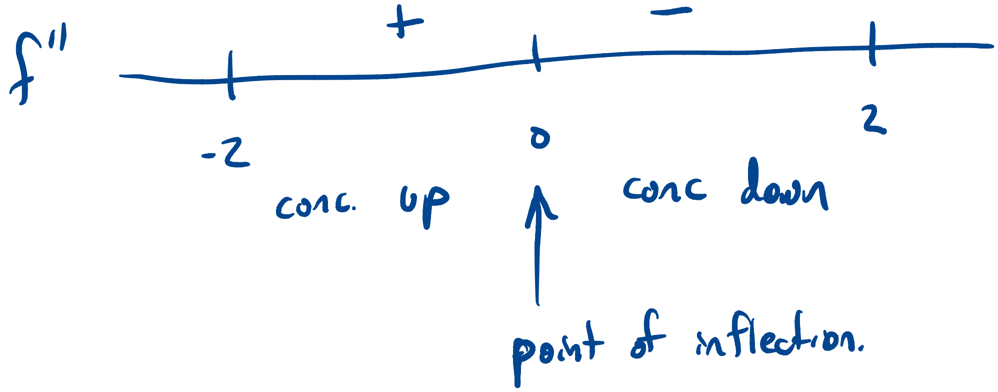
$f(\sqrt{2}) = f(-\sqrt{2}) = 0$

g. Classify each critical point as a local min/max/neither.

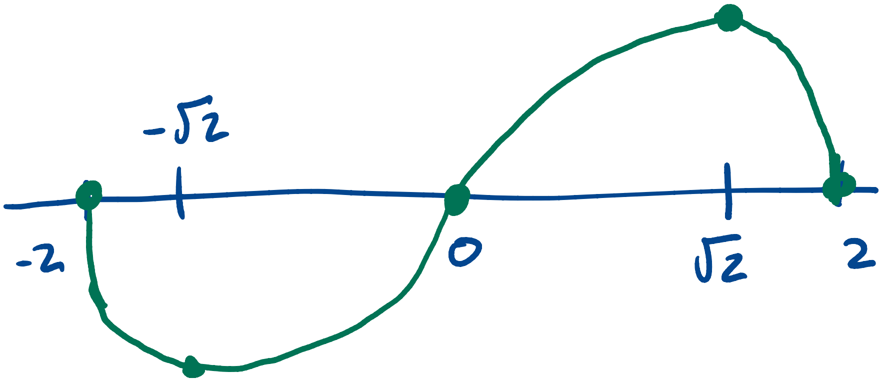


h. Find intervals where f is concave up/concave down and identify points of inflection

$$f''(x) = -2\frac{x^3}{(4-x^2)^{3/2}}$$



i. Sketch the graph of the function



3. Follow the guidelines from the previous worksheet to sketch the graph of

$$f(x) = \frac{x}{\sqrt{9+x^2}}.$$

a. What is the function's domain?

all real numbers

b. Does this function have any symmetry?

$$f(-x) = \frac{-x}{\sqrt{9+(-x)^2}} = -\frac{x}{\sqrt{9+x^2}} = -f(x) \Rightarrow \text{odd symmetry}$$

c. Find a few choice values of x to evaluate the function at.

$f(0) = 0$

d. What behaviour occurs for this function at $\pm\infty$?

$$\lim_{x\to\infty} \frac{x}{\sqrt{9+x^2}} = \lim_{x\to\infty} \frac{1}{\sqrt{9/x^2+1}} = 1 ; \lim_{x\to-\infty} \frac{x}{\sqrt{9+x^2}} = -1$$

e. Does the function have any vertical asymptotes? Where?

None

f. Find intervals where f is increasing/decreasing and identify critical points.

$$f'(x) = \frac{9}{(9+x^2)^{3/2}} > 0$$

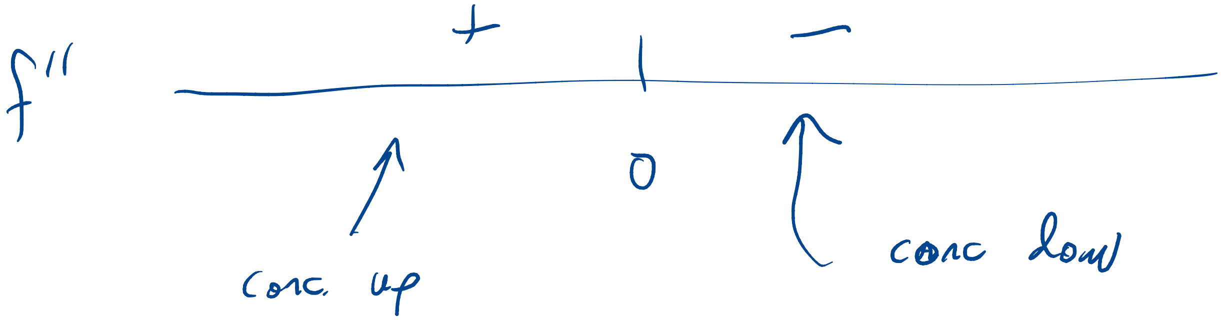
The function is always increasing.

g. Classify each critical point as a local min/max/neither.

None to classify.

h. Find intervals where f is concave up/concave down and identify points of inflection

$$f''(x) = -27(9+x^2)^{-5/2}x$$



i. Sketch the graph of the function