**L'Hôpital's Rule**
If $f$ and $g$ are differentiable and $g'(x) \neq 0$ on an interval containing $a$ (except possibly at $x = a$). If $\lim_{x\to a} f(x) = 0$ and $\lim_{x\to a} g(x) = 0$ then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

so long as the right-hand limit exists, or is $\pm\infty$. Moreover, the same technique can be used

- if $\lim_{x\to a} f(x) = \pm\infty$ and $\lim_{x\to a} g(x) = \pm\infty$,
- for one-sided limits,
- for limits at infinity.

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Compute the following limits.

**1.** $\lim_{x\to 0} \frac{\sin(5x)}{\sin(3x)}$ $\underset{0/0}{=} \lim_{x\to 0} \frac{5\cos(5x)}{3\cos(3x)} = \frac{5\cdot\cos(0)}{3\cdot\cos(0)} = \frac{5\cdot 1}{3\cdot 1} = \frac{5}{3}$

**2.** $\lim_{x\to 0} \frac{\cos(x) - 1}{x}$ $\underset{0/0}{=} \lim_{x\to 0} \frac{-\sin(x)}{1} = -\sin(0) = 0$

**3.** $\lim_{x\to 0} \frac{\cos(x)-1}{x^2}$

$$\underset{\frac{0}{0}}{=} \lim_{x\to 0} \frac{-\sin(x)}{2x} \underset{\frac{0}{0}}{=} \lim_{x\to 0} \frac{-\cos(x)}{2}$$

$$= \frac{-\cos(0)}{2} = -\frac{1}{2}$$

$\frac{-\infty}{\infty}$

**4.** $\lim_{x\to -\infty} \underbrace{xe^x}_{-\infty \cdot 0}$.

$$= \lim_{x\to -\infty} \frac{x}{1/e^x} = \lim_{x\to -\infty} \frac{x}{e^{-x}}$$

$$\underset{\frac{\infty}{\infty}}{=} \lim_{x\to -\infty} \frac{1}{-e^{-x}}$$

$$= \lim_{x\to -\infty} -e^x$$

$$= -0 = 0$$

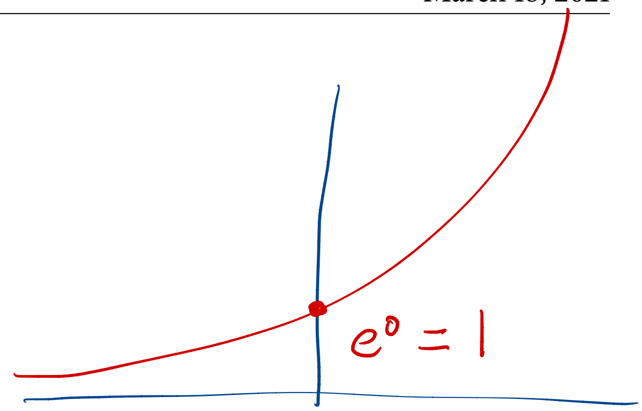
**5.** $\lim_{x\to 0} \frac{\arcsin(x)}{x}$

$$\lim_{x\to 0} \frac{\arcsin(x)}{x} \underset{\frac{0}{0}}{=} \lim_{x\to 0} \frac{\frac{1}{\sqrt{1-x^2}}}{1} = \lim_{x\to 0} \frac{1}{\sqrt{1-x^2}}$$

$$= \frac{1}{\sqrt{1-0^2}} = 1$$

**6.** $\lim_{x\to 0} \frac{e^x}{x+3}$. Careful!!

$$\lim_{x\to 0} \frac{e^x}{x+3} = \frac{e^0}{0+3} = \frac{1}{3}$$




**7.** $\lim_{x\to 0^+} \frac{e^{1/x}}{\ln x}$. $\underbrace{\phantom{x}}_{\frac{\infty}{-\infty}}$

$$= \lim_{x\to 0^+} \frac{e^{1/x}\cdot\left(-\frac{1}{x^2}\right)}{1/x} = \lim_{x\to 0^+} -\frac{x}{x^2} e^{1/x}$$

$$= \lim_{x\to 0^+} \underbrace{\frac{1}{x}}_{\infty} \underbrace{e^{1/x}}_{\infty}$$

$$= \infty$$

**8.** $\lim_{x\to\infty} \left(1 + \frac{5}{x}\right)^x$.

a) $$\lim_{x\to\infty} \ln\left(\left(1+\frac{5}{x}\right)^x\right) = \lim_{x\to\infty} x \ln\left(1+\frac{5}{x}\right)$$

$$= \lim_{x\to\infty} \frac{\ln(1+5/x)}{1/x}$$

$$\underset{\frac{0}{0}}{=} \lim_{x\to\infty} \frac{\frac{1}{1+5/x}\cdot\left(\frac{-5}{x^2}\right)}{\left(-1/x^2\right)} = \frac{5}{1+0} = 5$$

b) $\lim_{x \to \infty} \left(1 + \frac{5}{x}\right)^x = \lim_{x \to \infty} e^{\ln\left(\left(1+\frac{5}{x}\right)^x\right)}$

$$= e^{\lim_{x \to \infty} \ln\left(\left(1+5/x\right)^x\right)}$$

$$= \boxed{e^5}$$