First Derivative Test

Suppose f is a function with a derivative on (a, b), and if c is a point in the interval with $f'(c) = 0$.

- If $f'(x) > 0$ for x just to the left of c and $f'(x) < 0$ for x just to the right of c, then f has a local maximum at c.
- If $f'(x) < 0$ for x just to the left of c and $f'(x) > 0$ for x just to the right of c, then f has a local minimum at c.
- If $f'(c) = 0$ and $f'(x) < 0$ on both sides of c or $f'(x) > 0$ on both sides of c, then there is neither a local min nor a local max at c.

Second Derivative Test

Suppose f is a function with a continuous second derivative on (a, b), and that c is a point in the interval with $f'(c) = 0$.

- If $f''(c) > 0$ then f has a local minimum at c.
- If $f''(c) < 0$ then f has a local maximum at c.

Concave Up: $f'(x)$ increasing; $f''(x) > 0$

Concave Down: $f'(x)$ decreasing; $f''(x) < 0$

Point of Inflection: Value x where concavity changes; often $f''(x) = 0$

This worksheet considers the function

$$g(x) = x^2 e^x$$

1. Find all critical points of g.

2. Determine the intervals where g is increasing and where g is decreasing.

3. Use the First Derivative Test to classify each critical point as a local min/local max.

4. Determine the intervals where g is concave up and where f is concave down.

5. Find all points of inflection of g.

6. Use the Second Derivative Test to classify each critical point as a local min/local max (if possible).

7. Determine the value of g at each of its critical points.

8. Use the information determined thus far to sketch the graph of $g(x)$. You may used the fact, which we will justify next class, that $\lim_{x\to-\infty} f(x) = 0$.