First Derivative Test
Suppose f is a function with a derivative on (a, b), and if c is a point in the interval with $f'(c) = 0$.

- If $f'(x) > 0$ for x just to the left of c and $f'(x) < 0$ for x just to the right of c, then f has a local maximum at c.
- If $f'(x) < 0$ for x just to the left of c and $f'(x) > 0$ for x just to the right of c, then f has a local minimum at c.
- If $f'(c) = 0$ and $f'(x) < 0$ on both sides of c or $f'(x) > 0$ on both sides of c, then there is neither a local min nor a local max at c.

Second Derivative Test
Suppose f is a function with a continuous second derivative on (a, b), and that c is a point in the interval with $f'(c) = 0$.

- If $f''(c) > 0$ then f has a local minimum at c.
- If $f''(c) < 0$ then f has a local maximum at c.

Concave Up: $f'(x)$ increasing; $f''(x) > 0$

Concave Down: $f'(x)$ decreasing; $f''(x) < 0$

Point of Inflection: Value x where concavity changes; often $f''(x) = 0$

This worksheet considers the function

$$g(x) = x^2 e^x$$

number

1. Find all critical points of g.

Look for $g'(x) = 0$ or DNE

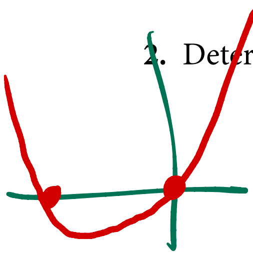
$g'(x) = 2x e^x + x^2 e^x$

$= (2x + x^2) e^x$

$= x(2+x) e^x$

$x = 0, \quad x = -2$

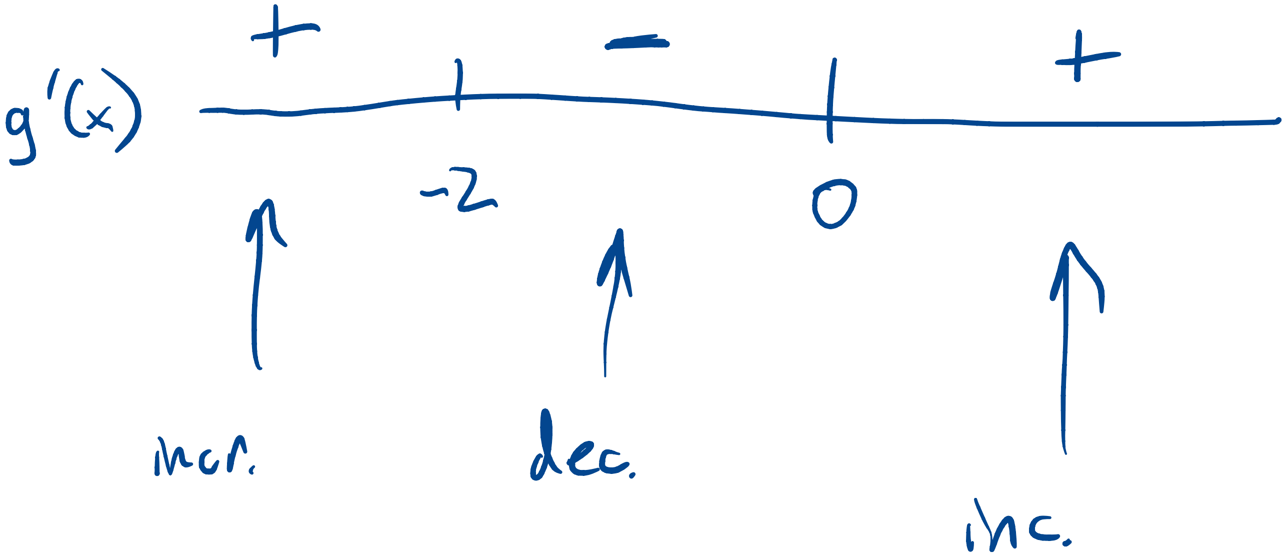
$-1\,(2+(-1))\,e^{-1}$



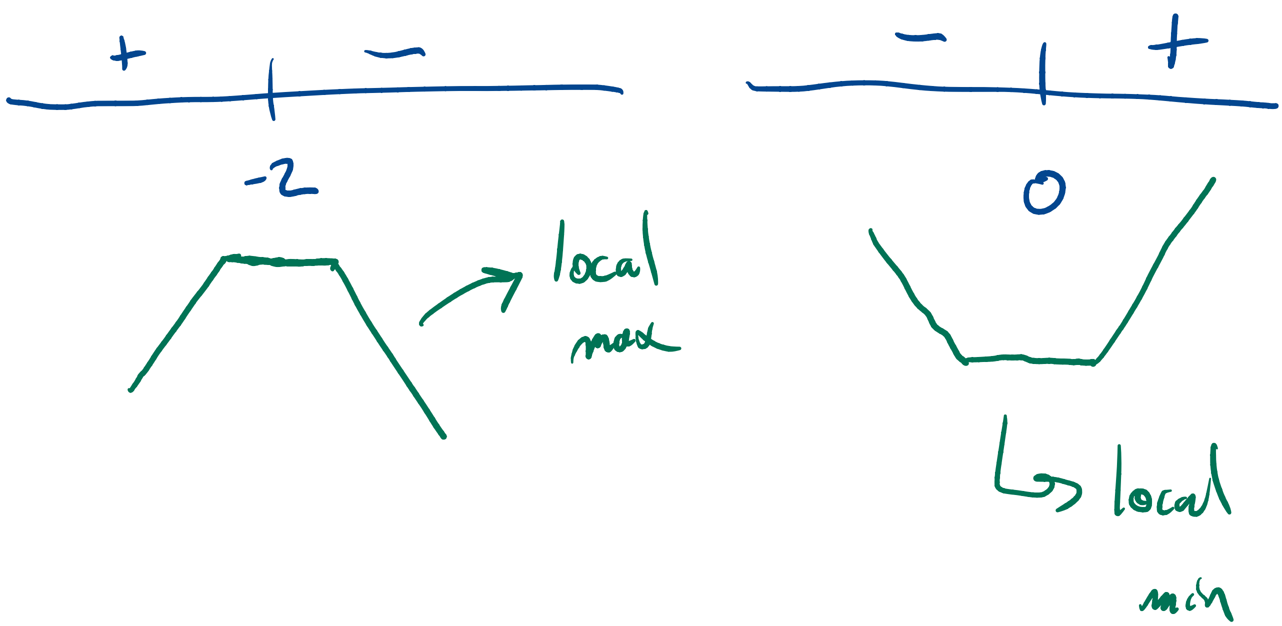
2. Determine the intervals where g is $\underbrace{\text{increasing}}_{g'(x)>0}$ and where g is $\underbrace{\text{decreasing}}_{g'(x)<0}$.

$g'(x) = \underbrace{x(2+x)}e^x$

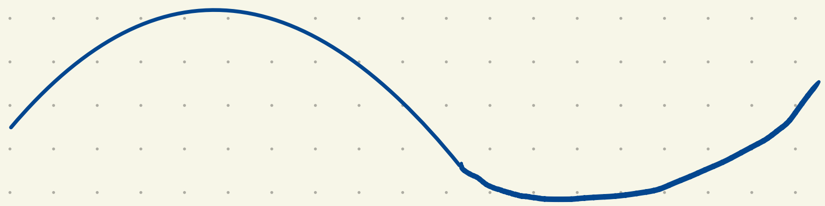
$(x^2+2x)e^x$



3. Use the First Derivative Test to classify each critical point as a local min/local max.



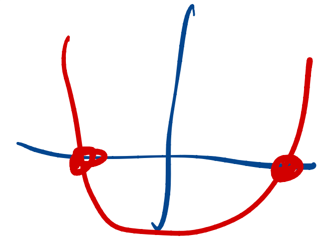

+
−
+
-2
0



4. Determine the intervals where g is concave up and where ~~f~~ g is concave down.

$g''(x) > 0$

$g''(x) < 0$



$g'(x) = (2x + x^2)e^x$

$$g''(x) = (2 + 2x)e^x + (2x + x^2)e^x = (2 + 4x + x^2)e^x$$

analyze this!

5. Find all points of inflection of g.

g'': $+$ | $-2-\sqrt{2}$ | $-$ | $-2+\sqrt{2}$ | $+$

$x = -2 \pm \sqrt{2}$

$+$ | $-2-\sqrt{2}$ | $-$ | $-2+\sqrt{2}$ | $+$

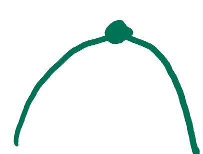
6. Use the Second Derivative Test to classify each critical point as a local min/local max (if possible).

g': $+$ | -2 | $-$ | 0 | $+$

g'': $+$ | $-2-\sqrt{2}$ | $-$ | $-2+\sqrt{2}$ | $+$

local min

local max



7. Determine the value of g at each of its critical points.

$x = 0, -2$

$g(x) = x^2 e^x$

$g(0) = 0^2 e^0 = 0$

$g(-2) = 4e^{-2} \quad (>0)$

$g(x) = x^2 e^x$

8. Use the information determined ~~thus far to sketch the graph~~ of $g(x)$. You may used the fact, which we will justify next class, that $\lim_{x\to-\infty} f(x) = 0$.

$\lim_{x\to\infty} x^2 e^x = \infty$

$\lim_{x\to-\infty} x^2 e^x = \lim_{x\to-\infty} \frac{x^2}{e^{-x}} = \lim_{x\to-\infty} \frac{2x}{-e^{-x}}$

$\infty \cdot 0$

$\frac{\infty}{\infty}$

$\underset{\frac{\infty}{\infty}}{=} \lim_{x\to-\infty} \frac{2}{e^{-x}}$

$= \lim_{x\to-\infty} 2e^x$

$= 0$

$$\frac{1}{e^{-x}} = e^x$$

$$\frac{1}{(e^x)^{-1}} = \left((e^x)^{-1}\right)^{-1} = e^x$$

x^2e^x