Vocabulary

Suppose $f(x)$ is a real-valued function with domain D and suppose c is a point in D.

1. $f(c)$ is an **absolute maximum value** for f if $f(c) \geq f(x)$ for each x in D.
2. $f(c)$ is a **(absolute) minimum value** for f if $f(c) \leq f(x)$ for each x in D.
3. $f(c)$ is a **local maximum value** for f if $f(c) \geq f(x)$ for each x in D *near* c.
4. $f(c)$ is a **local minimum value** for f if $f(c) \leq f(x)$ for each x in D *near* c.
5. We say c is a **critical point** for f if either $f'(c) = 0$ or $f'(c)$ does not exist.

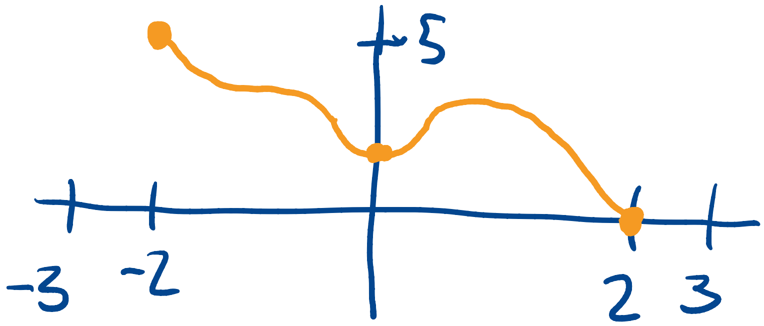
$f(x)=x$

Key Tools

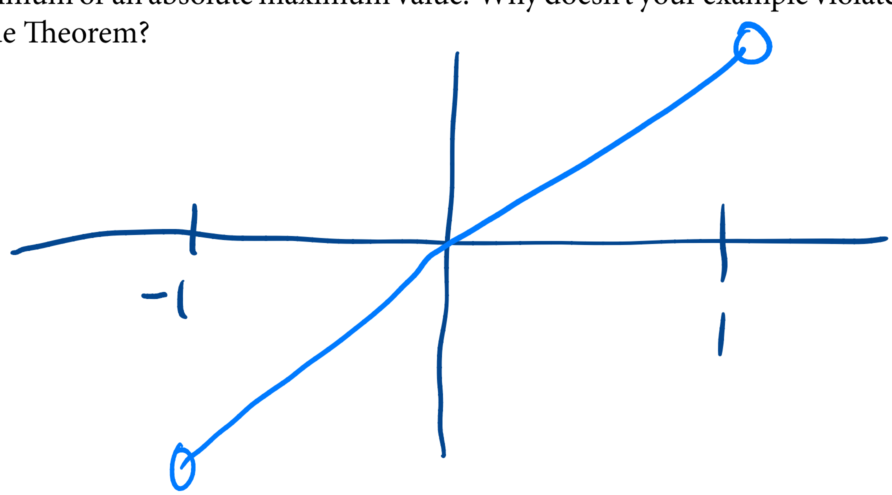
1. [Fermat's Theorem] If $f(c)$ is a (local or absolute) maximum/minimum value, and if f is defined on both sides of c, and if $f'(c)$ exists, then $f'(c) = 0$.
2. [Extreme Value Theorem] If the domain of f is a closed, bounded interval, and if f is continuous, then f is guaranteed to have both a maximum and a minimum value.

$\to [a,b]$

1. Sketch the graph of a function with domain $[-3, 3]$ that has an absolute maximum of 5 at $x = -2$, an absolute minimum of 0 at $x = 2$ and a local minimum of 2 at $x = 0$ that is not an absolute minimum.

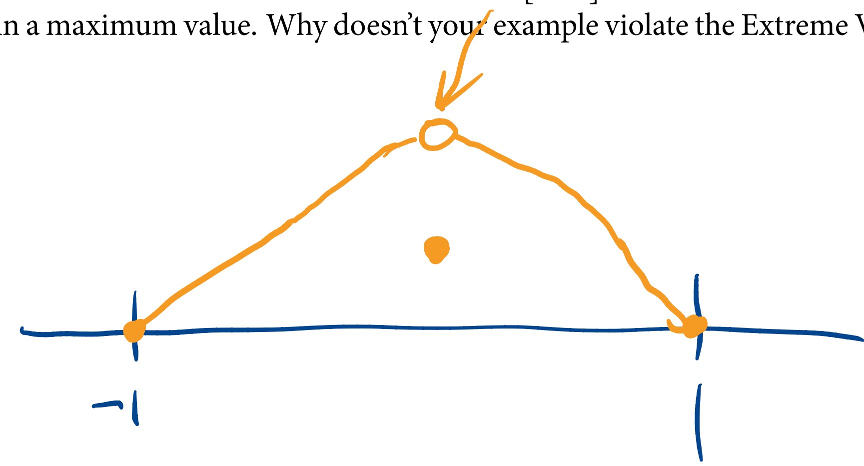


2. Give an example of a function with domain $(-1, 1)$ that does not attain either an absolute minimum or an absolute maximum value. Why doesn't your example violate the Extreme Value Theorem?

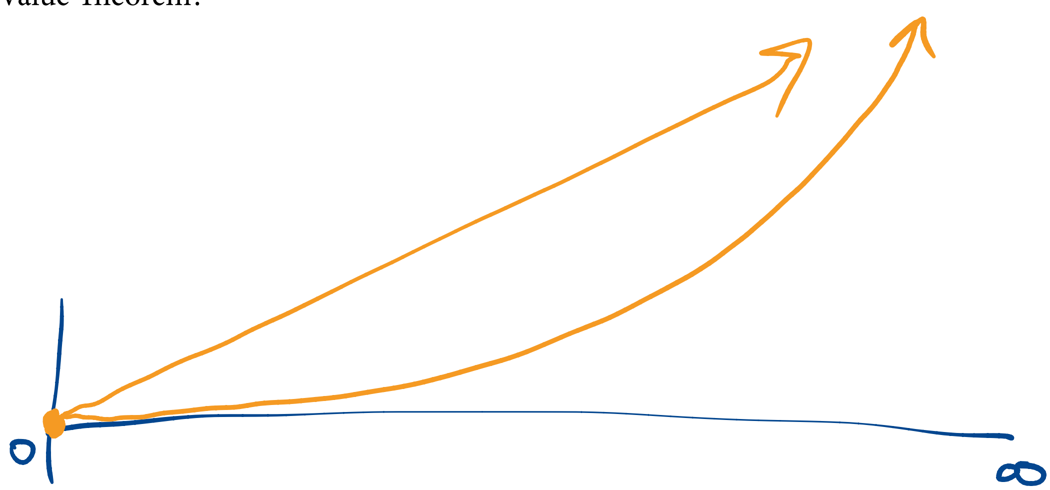


$f(x)=x$

3. Sketch a discontinuous function with domain $[-1, 1]$ that attains a minimum but does not attain a maximum value. Why doesn't your example violate the Extreme Value Theorem?



4. Give an example of a continuous function with domain $[0, \infty)$ that attains a minimum but does not attain a maximum value. Why doesn't your example violate the Extreme Value Theorem?



5. Consider the function $\sec(x)$. Sketch this function. From the sketch answer the following. Does this function have any absolute maximums? Absolute minimums? Local maximums? Local minimums?

6. Find all critical points of the function $f(x) = \sin(x)^{1/3}$.

7. Key Tool: Closed Interval Method

To find a maximum or minimum value for a continuous function defined on an closed, bounded interval $[a, b]$, look in all of the following locations:

1. The end points.
2. The critical points. $f'(x) = 0$ or $f'(x)$ does not exist.

Find the absolute maximum and minimum values of $f(x) = x - x^{1/3}$ on the interval $[-1, 4]$, and the locations where those values are attained.

Need to look at a) end points

b) critical points

$f'(x) = 0$ or DNE

$$f'(x) = 1 - \frac{1}{3}x^{-2/3} = 1 - \frac{1}{3}\left(\frac{1}{x}\right)^{2/3}$$

↳ $x=0$ is a no-go!

$$1 - \frac{1}{3}x^{-2/3} = 0$$

$$x^{-2/3} = 3$$

$$\left(x^{-2/3}\right)^{-3/2} = 3^{-3/2}$$

$$x = 3^{-3/2}$$

$$x^{-2/3} = 3$$

$$\left(\left(\frac{1}{x}\right)^2\right)^{1/3} = 3$$

$$\left(\frac{1}{x}\right)^2 = 3^3$$

$$\frac{1}{x} = \pm\sqrt{3^3} = \pm 3^{3/2}$$

$$\underbrace{x = \pm 3^{-3/2}}$$

two critical points

where $f'(x) = 0$

One critical point, $x = 0$

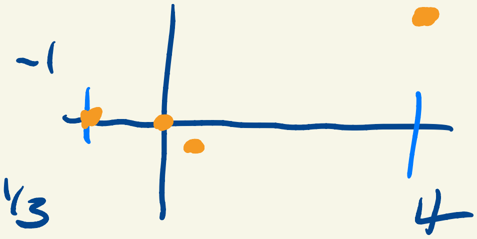
where $f'(x)$ DNE

Three critical points: $x = 0$

$$x = 3^{-3/2} = \left(\frac{1}{3}\right)^{3/2}$$

$$x = -3^{-3/2}$$

Two end points: -1, 4



$x - x^{1/3}$

$$f(0) = 0$$

$$f\left(\left(\tfrac{1}{3}\right)^{3/2}\right) = -0.3849\ldots$$

$$f(-1) = -1 - (-1)^{1/3} = -1 + 1 = 0$$

$$f(4) = 4 - 4^{1/3} = 2.41$$

$$f\left(-\left(\frac{1}{3}\right)^{3/2}\right) = +0.3849$$

Minimum of -0.3849 at $x = \left(\frac{1}{3}\right)^{3/2}$.

Maximum of 2.41 at $x = 4$

8. Find the absolute maximum and minimum values of $f(x) = e^{-x^2}$ on the interval $[-2, 3]$, and the locations where those values are attained.

9. Find the maximum and minimum values of $f(x) = x - x^{1/3}$ on the interval [-1,4]. Determine where those maximum and minimum values occur.

10. Find the maximum and minimum values of $f(x) = x + \frac{1}{x}$ on the interval [1/5,4]. Determine where those maximum and minimum values occur.

Need to look at 1) endpoints

2) critical points

$f'(x) = 1 - \frac{1}{x^2}$

$f'(x) = 0$

$1 - \frac{1}{x^2} = 0$

$1 = \frac{1}{x^2}$ $x^2 = 1$

$x = \pm 1$

No spots where $f'(x)$ DNE in $[1/5, 4]$

One spot where $f'(x) = 0$ in - - - .

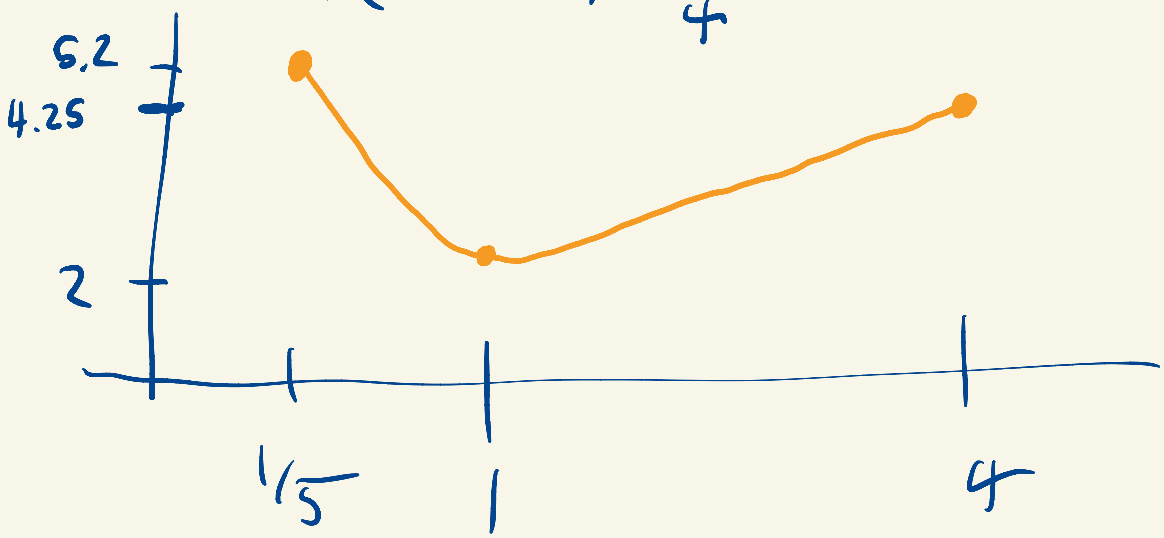
$\hookrightarrow x = 1$

Check at $\qquad f(x) = x + \frac{1}{x}$

$x = \frac{1}{5} \qquad f(1/5) = \frac{1}{5} + 5 = 5.2$ max value

$x = 1 \qquad f(1) = 1 + \frac{1}{1} = 1 + 1 = 2$ min value

$x = 4 \qquad f(4) = 4 + \frac{1}{4} = 4.25$



11. Find the maximum and minimum values of $f(x) = x^{2/3}$ on the interval [-8,8]. Determine where those maximum and minimum values occur.

12. A ball thrown in the air at time $t = 0$ has a height given by

$$h(t) = h_0 + v_0 t - \frac{1}{2} g_0 t^2$$

meters where t is measured in seconds, h_0 is the height at time 0, v_0 is the velocity (in meters per second) at time 0 and g_0 is the constant acceleration due to gravity (in m/s^2). Assuming $v_0 > 0$, find the time that the ball attains its maximum height. Then find the maximum hight.