1. I'm tired of doing all the work around here. It's your turn. You're going to show that

$$\frac{d}{dx}\ln(x) = \frac{1}{x}.$$

Start with the equation $y = \ln(x)$.

1. Solve this equation for x.

$$y = \ln(x) \longrightarrow e^y = e^{\ln(x)} = x$$

2. Take an implicit derivative with respect to x, and solve for dy/dx.

$$e^y = x$$

$$\frac{d}{dx} e^y = \frac{d}{dx} x \longrightarrow e^y \cdot y' = 1$$

$$y' = \frac{1}{e^y}$$

3. Now convert dy/dx into an expression that only involves x. (Tah dah!)

$$\frac{dy}{dx} = \frac{1}{x} \qquad \frac{d}{dx}\ln(x) = \frac{1}{x}$$

2. Compute $\frac{d}{dx}\ln\left(x + e^{3x}\right)$.

$$\frac{d}{dx}\ln(x+e^{3x}) = \frac{1}{x+e^{3x}} \cdot \frac{d}{dx}(x+e^{3x}) = \frac{1}{x+e^{3x}} \cdot \left(1 + \frac{d}{dx}e^{3x}\right)$$

$$= \frac{1}{x+e^{3x}} \cdot (1+3e^{3x})$$

3. Compute $\frac{d}{dx}\ln(\cos(x))$ and simplify your expression.

$$\frac{d}{dx}\ln(\cos(x)) = \frac{1}{\cos(x)} \cdot \frac{d}{dx}(\cos(x)) = -\frac{\sin(x)}{\cos(x)} = -\tan(x)$$

$$\frac{d}{dx} e^{3x} = e^{3x} \cdot \frac{d}{dx}(3x)$$

$$= 3e^{3x}$$

inside: $3x$

outside: e^{x}

outside': $e^{\text{☺}}$

4. How can we compute $\frac{d}{dx}5^x$?

1. Rewrite $5^x = e^{ax}$ for a certain constant a. Your job is to find a!

$5^x = e^{ax}$ | $\ln(5^x) = \ln(e^{ax}) \to x\ln(5) = ax\ln(e)$

$\boxed{a = \ln(5)}$

2. Now compute $\frac{d}{dx}5^x$ by taking the derivative of e^{ax} instead.

$$\frac{d}{dx}5^x = \frac{d}{dx}e^{\ln 5\, x} = \ln(5)\, e^{\ln(5)x}$$

3. Rewrite your previous answer so that the letter e does not appear.

$\boxed{e^{\ln(5)x} = 5^x}$

$$\frac{d}{dx}5^x = \ln(5)\, 5^x$$

5. Derive a formula for $\frac{d}{dx}\log_5(x)$. You can either use a change of base formula, or you can repeat the technique used to find the derivative of $\ln(x)$. Heck, do it both ways.

$y = \log_5(x)$ $\qquad e^{\ln(x)} = x$

$5^y = 5^{\log_5(x)} = x$

(want y')

$5^y = x$

$\frac{d}{dx}(5^y) = \frac{d}{dx}x$

$\ln(5)\, 5^y \cdot y' = 1 \quad \to \quad y' = \frac{1}{\ln(5)\, 5^y}$

$y' = \frac{1}{\ln(5)\, x}$

$$\boxed{\frac{d}{dx}\log_5(x) = \frac{1}{\ln(5)\, x}}$$

6. Suppose you wish to differentiate

$$f(x) = x^x.$$

The tool to use is called logarithmic differentiation.

Start with the equation $y = x^x$. $\longrightarrow$ want $\frac{dy}{dx}$, y'

1. Apply the natural logarithm to both sides of the equation and simplify.

$$\ln(y) = \ln(x^x) = x\ln(x)$$

2. Take an implicit derivative with respect to x, and solve for dy/dx.

$$\ln(y) = x\ln(x)$$

$$\frac{d}{dx}(\ln(y)) = \frac{d}{dx}(x\ln(x)) \longrightarrow \frac{1}{y}\cdot y' = 1\cdot\ln(x) + x\cdot\frac{1}{x}$$

$$y' = y(\ln(x) + 1)$$

3. Now convert dy/dx into an expression that only involves x. (Tah dah!)

$$y' = x^x(\ln(x) + 1)$$

7. Differentiate $f(x) = x^{\sin(x)}$.

$$y = x^{\sin(x)} \qquad \text{want } y'$$

$$\ln(y) = \ln\left(x^{\sin(x)}\right) = \sin(x)\ln(x)$$

$$\frac{1}{y}y' = \cos(x)\ln(x) + \frac{\sin(x)}{x} \Rightarrow y' = x^{\sin(x)}\left[\cos(x)\ln(x) + \frac{\sin(x)}{x}\right]$$

8. We wish, for whatever bizarre reason, to compute dy/dx if

$$y = \frac{(x^2+1)(x+3)^{1/2}}{x-1}.$$

One can use the product and quotient rules. But logarithmic differentiation can be a useful tool instead. known as logarithmic differentiation.

1. Take the natural logarithm of both sides of the equation.

$$\ln(y) = \ln\left[\frac{(x^2+1)(x+3)^{1/2}}{x-1}\right]$$

$$\frac{d}{dx}(x-1) = \frac{d}{dx}x - \frac{d}{dx}1$$

2. Use log rules such as $\ln(AB) = \ln(A) + \ln(B)$ to expand the right-hand side of this equation

$$\ln(y) = \ln(x^2+1) + \ln((x+3)^{1/2}) - \ln(x-1)$$
$$= \ln(x^2+1) + \frac{1}{2}\ln(x+3) - \ln(x-1)$$

3. Compute (implicitly) dy/dx and solve for dy/dx.

$$\frac{1}{y}y' = \frac{1}{x^2+1}\cdot 2x + \frac{1}{2}\frac{1}{x+3}\cdot 1 - \frac{1}{x-1}\cdot 1$$

4. Convert the expression for dy/dx so that it only involves x, and there are no appearances of y.

$$y' = y\cdot\left[\frac{2x}{x^2+1} + \frac{1}{2}\frac{1}{x+3} - \frac{1}{x-1}\right]$$

$$= \frac{(x^2+1)(x+3)^{1/2}}{x-1}\cdot\left[\frac{2x}{x^2+1} + \frac{1}{2}\frac{1}{x+3} - \frac{1}{x-1}\right]$$