

1. Find dy/dx if $y \cos(x) = x^2 + y^2$

$$y' \cos(x) - y \sin(x) = 2x + 2y y'$$

$$y' [\cos(x) - 2y] = 2x + y \sin(x)$$

$$y' = \frac{2x + y \sin(x)}{\cos(x) - 2y}$$

$(-1, 27)$

2. Show that $(-1, 27)$ lies on the asteroid $x^{2/3} + y^{2/3} = 10$. Then compute dy/dx at that point.

$$(-1)^{2/3} + (27)^{2/3} = 1 + 3^2 = 10$$

$$\frac{d}{dx} (x^{2/3} + y^{2/3}) = \frac{d}{dx} 10$$

$$\frac{2}{3} x^{-1/3} + \frac{2}{3} y^{-1/3} y' = 0$$

$$y' = - \frac{x^{-1/3}}{y^{-1/3}} = - \frac{y^{1/3}}{x^{1/3}}$$

at $(x, y) = (-1, 27)$

$$y' = - \frac{(27)^{1/3}}{(-1)^{1/3}} = 3$$

3. Find dy/dx if $y = \arcsin(3x)$.

$$\begin{aligned} \frac{d}{dx} \arcsin(3x) &= \frac{1}{\sqrt{1-(3x)^2}} \cdot \frac{d}{dx}(3x) \\ &= \frac{3}{\sqrt{1-(3x)^2}} \end{aligned}$$

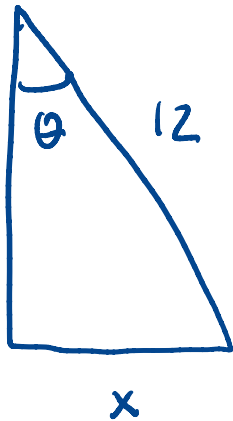
$$\frac{d}{dx} \arctan(x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx} \arcsin(x) = \frac{1}{\sqrt{1-x^2}}$$

4. Find dy/dx if $y = \arctan(\sqrt{4-x^2})$.

$$\begin{aligned} \frac{d}{dx} \arctan(\sqrt{4-x^2}) &= \frac{1}{1+4-x^2} \cdot \frac{d}{dx} \sqrt{4-x^2} \\ &= \frac{1}{5-x^2} \cdot \frac{1}{2\sqrt{4-x^2}} \cdot \frac{d}{dx} \underbrace{(4-x^2)}_{\rightarrow -2x} \\ &= \frac{1}{5-x^2} \cdot \frac{-x}{\sqrt{4-x^2}} \end{aligned}$$

5. A 12-foot ladder is leaning against a wall. Let x denote the distance of the base of the ladder from the wall, and let θ be the angle between the ladder and the wall. How fast does the angle θ change with respect to x ?



$$x = 12 \sin \theta$$

$$\arcsin\left(\frac{x}{12}\right) = \theta$$

$$\frac{d\theta}{dx} = \frac{1}{12} \frac{1}{\sqrt{1 - (x/12)^2}} = \frac{1}{\sqrt{12^2 - x^2}}$$

6. I compute that $d\theta/dx \approx 0.1$ when $x = 7$. What does this mean in language your parents can understand? Feel free to express your answer in terms of degrees instead of radians.

When the base of the ladder is 7 ft from the wall, the angle at the wall changes at a rate of

0.1 $\frac{\text{rad}}{\text{foot}}$ as I change the distance of the base of

the ladder from the wall. Note: $0.1 \frac{\text{rad}}{\text{foot}} = 0.1 \cdot \frac{360}{2\pi} \frac{\text{deg}}{\text{foot}}$

$$\approx 5^\circ/\text{foot}$$