1. The volume of a snowball of radius r is $V(r) = (4/3)\pi r^3$, where r is measured in inches and V is in measured in inches cubed. Explain what $V'(2) \approx 50.265$ means in language your parents could understand.

"As the radius of a snowball with 2 inch radius increases the volume of the snowball increases at the rate of 50,265 $\frac{\text{inches}^3}{\text{inch}}$!"

2. Compute $\frac{d}{dx}\cot(x)$

$$\boxed{\frac{d}{dx}\cot(x)} = \frac{d}{dx}\frac{\cos(x)}{\sin(x)} = \frac{\left(\frac{d}{dx}\cos(x)\right)\sin(x) - \cos(x)\cdot\frac{d}{dx}\sin(x)}{\sin^2(x)}$$

$$= \frac{-\sin^2(x) - \cos^2(x)}{\sin^2(x)} = \frac{-1\left(\sin^2(x)+\cos^2(x)\right)}{\sin^2(x)}$$

$$= \frac{-1}{\sin^2(x)}$$

$$= \boxed{-\csc^2(x)}$$

3. Compute $\frac{d}{dx}\sec(x)$

$$\boxed{\frac{d}{dx}\sec(x)} = \frac{d}{dx}\frac{1}{\cos(x)} = \frac{-\frac{d}{dx}\cos(x)}{\cos^2(x)}$$

$$= \frac{\sin(x)}{\cos^2(x)}$$

$\sec(x)\tan(x)$

$$= \frac{\sin(x)}{\cos(x)}\cdot\frac{1}{\cos(x)} = \boxed{\tan(x)\sec(x)}$$

4. Compute the second derivative $\frac{d^2}{dx^2}e^x\cos(x)$

Start with just one derivative:

$$\frac{d}{dx}e^x\cos(x) = \left(\frac{d}{dx}e^x\right)\cos(x) + e^x\frac{d}{dx}\cos(x)$$

$$= e^x\cos(x) - e^x\sin(x)$$

$$= e^x\left[\cos(x) - \sin(x)\right]$$

Then

$$\frac{d^2}{dx^2}e^x\cos(x) = \frac{d}{dx}e^x\left[\cos(x) - \sin(x)\right]$$

$$= \left(\frac{d}{dx}e^x\right)\left[\cos(x) - \sin(x)\right] + e^x\frac{d}{dx}\left[\cos(x) - \sin(x)\right]$$

$$= e^x\left[\cos(x) - \sin(x)\right] + e^x\left[-\sin(x) - \cos(x)\right]$$

$$= \boxed{-2e^x\sin(x)}$$

5. Find the equation of the tangent line of the graph of $y = \sin(x)$ at $x = \pi/3$.

point: $(\pi/3, \sin(\pi/3)) = \left(\frac{\pi}{3}, \frac{\sqrt{3}}{2}\right)$

slope: $y'(x) = \frac{d}{dx}\sin(x) = \cos(x)$

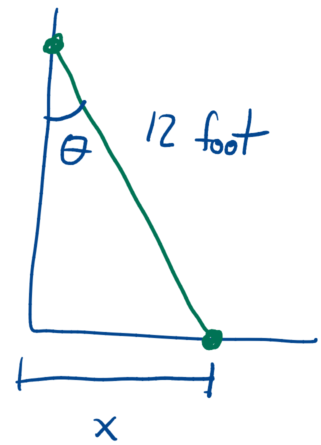
$y'(\pi/3) = \cos(\pi/3) = \frac{1}{2}$

Point-slope form: $(y = y_0 + m(x - x_0))$

$$y = \frac{\sqrt{3}}{2} + \frac{1}{2}\left(x - \frac{\pi}{3}\right)$$

6. A 12 foot ladder rests against a wall. Let θ be the angle between the ladder and the wall and let x be the distance from the base of the ladder and the wall.

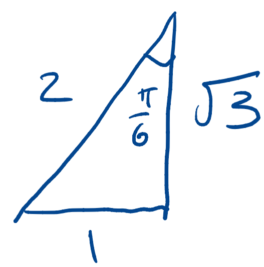
a. Compute x as a function of θ.



12, x, θ

$$\frac{x}{12} = \sin\theta$$

$$x = 12\sin\theta$$



b. How fast does x change with respect to θ when $\theta = \pi/6$? Include units in your answer.

$$\frac{dx}{d\theta} = \frac{d}{d\theta} 12\sin\theta = 12\frac{d}{d\theta}\sin\theta = 12\cdot\cos\theta$$

$$\left.\frac{dx}{d\theta}\right|_{\theta=\frac{\pi}{6}} = 12\cos\left(\pi/6\right) = 12\cdot\frac{\sqrt{3}}{2} = 6\sqrt{3}$$

$$\approx 10\ \frac{\text{feet}}{\text{radians}}$$