

1. A rocket is launching, and its height h in meters is a function of t in seconds (so we are considering the function $h(t)$). Explain what $h'(10) = 1035$ means in language your parents could understand. Your answer must include units.

The rocket is rising at a rate of $1035 \frac{\text{m}}{\text{s}}$.

velocity: r.o.c. of position

Compute derivatives of the following functions using derivative rules.

2. $f(t) = e^t \cos(t)$

$$f'(t) = \left(\frac{d}{dt} e^t \right) \cdot \cos(t) + e^t \frac{d}{dt} \cos(t)$$

$$= e^t \cdot \cos(t) + e^t (-\sin(t))$$

$$= e^t [\cos(t) - \sin(t)]$$

3. $f(x) = \frac{x}{1+e^x}$

$$\frac{d}{dx} \frac{x}{1+e^x} = \frac{\left(\frac{d}{dx} x \right) \cdot (1+e^x) - x \frac{d}{dx} (1+e^x)}{(1+e^x)^2}$$

$$= \frac{1 \cdot (1+e^x) - x e^x}{(1+e^x)^2} = \frac{1+e^x - x e^x}{(1+e^x)^2}$$

4. $f(t) = e^{-t}$

$$\begin{aligned}\frac{d}{dt} e^{-t} &= \frac{d}{dt} \frac{1}{e^t} = - \frac{\frac{d}{dt} (e^t)}{(e^t)^2} = - \frac{e^t}{(e^t)^2} \\ &= - \frac{1}{e^t} \\ &= \boxed{-e^{-t}}\end{aligned}$$

5. $e^{-t} \cos(t)$

$$\begin{aligned}\frac{d}{dt} (e^{-t} \cos(t)) &= \frac{d}{dt} e^{-t} \cdot \cos(t) + e^{-t} \cdot \frac{d}{dt} (\cos(t)) \\ &= (-e^{-t}) \cdot \cos(t) + e^{-t} (-\sin(t)) \\ &= \boxed{-e^{-t} [\cos(t) + \sin(t)]}\end{aligned}$$

6. $f(x) = \frac{1}{1+x^2}$

$$\frac{df}{dx} = \frac{d}{dx} \left(\frac{1}{1+x^2} \right) = \frac{-\frac{d}{dx}(1+x^2)}{(1+x^2)^2}$$

$$= \boxed{\frac{-2x}{(1+x^2)^2}}$$

7. $f(x) = (1+x^2)e^x \sin(x)$

$$\frac{d}{dx} \left[(1+x^2)e^x \sin(x) \right] = \frac{d}{dx} \left[(1+x^2)e^x \right] \cdot \sin(x) + (1+x^2)e^x \cdot \frac{d}{dx} \sin(x)$$

$$= \left[\left(\frac{d}{dx}(1+x^2) \right) e^x + (1+x^2) \frac{d}{dx} e^x \right] \cdot \sin(x) + (1+x^2)e^x \cos(x)$$

$$= \boxed{\left[2xe^x + (1+x^2)e^x \right] \cdot \sin(x) + (1+x^2)e^x \cos(x)}$$

$$8. f(v) = \left(1 + \frac{1}{v}\right) \left(2 - \frac{1}{v}\right)$$

$$(1 + v^{-1})(2 - v^{-1}) = 2 + v^{-1} - v^{-2}$$

$$\begin{aligned} \frac{d}{dv} f(v) &= \frac{d}{dv} [2 + v^{-1} - v^{-2}] = 0 + (-1)v^{-2} - (-2)v^{-3} \\ &= -v^{-2} + 2v^{-3} \end{aligned}$$

$$9. f(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

$$\frac{d}{d\theta} \frac{\sin \theta}{\cos \theta} = \frac{\left(\frac{d}{d\theta} \sin \theta\right) \cdot \cos \theta - \sin \theta \frac{d}{d\theta} (\cos \theta)}{\cos^2 \theta}$$

$$= \frac{\cos^2 \theta - \sin \theta \cdot (-\sin \theta)}{\cos^2 \theta}$$

$$= \frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta}$$

$$\frac{d}{d\theta} \tan \theta = \sec^2 \theta$$

← Just showed this!