Compute derivatives of the following functions using derivative rules.

1. $f(x) = (x-2)(2x+3)$ $= 2x^2 - x - 6$

$$f'(x) = \frac{d}{dx}\left[2x^2 - x - 6\right] = 2\frac{d}{dx}x^2 - \frac{d}{dx}x - \frac{d}{dx}6 = \boxed{4x - 1}$$

2. $f(t) = \sqrt{t} - e^t$ $= t^{1/2} - e^t$

$$f'(t) = \frac{d}{dt}\left(t^{1/2} - e^t\right) = \boxed{\frac{1}{2}t^{-1/2} - e^t}$$

3. $f(x) = \dfrac{x^2 + x - 1}{\sqrt{x}}$ $= x^{3/2} + x^{1/2} - x^{-1/2}$

$$f'(x) = \frac{d}{dx}\left[x^{3/2} + x^{1/2} - x^{-1/2}\right] = \boxed{\frac{3}{2}x^{1/2} + \frac{1}{2}x^{-1/2} + \frac{1}{2}x^{-3/2}}$$

4. $V(r) = \dfrac{4}{3}\pi r^3$

$$V'(r) = \frac{d}{dr}\left[\frac{4}{3}\pi r^3\right] = \frac{4\pi}{3}\frac{d}{dr}r^3 = \frac{4\pi}{3}3r^2$$

$$= \boxed{4\pi r^2}$$

5. $f(x) = e^{x-3}$ $= e^x \cdot e^{-3}$

$$f'(x) = \frac{d}{dx}e^x \cdot e^{-3} = e^{-3}\frac{d}{dx}e^x = e^{-3}e^x = \boxed{e^{x-3}}$$

6. Use the definition of the derivative to show $\frac{d}{dx}x^3 = 3x^2$.

$$\frac{d}{dx}x^3 = \lim_{h\to 0}\frac{(x+h)^3 - x^3}{h} = \lim_{h\to 0}\frac{x^3+3x^2h+3xh^2+h^3-x^3}{h}$$

$$= \lim_{h\to 0}\frac{3x^2h+3xh^2+h^3}{h}$$

$$= \lim_{h\to 0} 3x^2+3xh+h^2$$

$$= 3x^2+3x\cdot 0+0^2$$

$$\boxed{= 3x^2}$$

7. Use the definition of the derivative to show $\frac{d}{dx}x^{-1} = (-1)x^{-2}$.

In class Thursday!

8. Estimate $f'(0)$ to three decimal digits if $f(x) = 3^x$

$$f'(0) = \lim_{h\to 0}\frac{3^h - 3^0}{h} = \lim_{h\to 0}\frac{3^h-1}{h}$$

h	$(3^h-1)/h$
0.1	1.161 - - - - -
0.01	1.1046 - - - -
0.001	1.09867 - - - -
0.0001	1.09861 - - - -

$f'(0) \approx 1.0986$