1. Compute

$$\lim_{x\to-\infty} \frac{x^2+1}{x^2-1}. \approx \frac{x^2}{x^2} \approx 1$$

$$\lim_{x\to-\infty} \frac{x^2+1}{x^2-1} = \lim_{x\to-\infty} \frac{\frac{1}{x^2}}{1/x^2} \frac{x^2+1}{x^2-1}$$

$$= \lim_{x\to-\infty} \frac{1+1/x^2}{1-1/x^2}$$

$$= \frac{\lim_{x\to-\infty} 1+1/x^2}{\lim_{x\to-\infty} 1-1/x^2} = \frac{1+0}{1-0} \boxed{=1}$$

2. Compute

$$\lim_{x\to-\infty} \frac{\sqrt{1+2x^4}}{2-x^2}.$$

$$\lim_{x\to-\infty} \frac{\sqrt{1+2x^4}}{2-x^2} = \lim_{x\to-\infty} \frac{\frac{1}{x^2}}{1/x^2} \frac{\sqrt{1+2x^4}}{2-x^2}$$

$$= \lim_{x\to-\infty} \frac{\sqrt{\frac{1}{x^4}+2\frac{x^4}{x^4}}}{2/x^2-1} \qquad \boxed{-\sqrt{2}}$$

$$= \lim_{x\to-\infty} \frac{\sqrt{1/x^4+2}}{2/x^2-1} = \frac{\sqrt{0+2}}{-1}$$

3. Compute

$$\lim_{x\to\infty} \sqrt{9x^2+1} - 3x.$$

Hint: Multiply by $1 = \dfrac{\sqrt{9x^2+1}+3x}{\sqrt{9x^2+1}+3x}$.

$$\lim_{x\to\infty} \sqrt{9x^2+1} - 3x = \lim_{x\to\infty} \left(\sqrt{9x^2+1} - 3x\right) \frac{\sqrt{9x^2+1}+3x}{\sqrt{9x^2+1}+3x}$$

$$= \lim_{x\to\infty} \frac{1}{\sqrt{9x^2+1}+3x}$$

$$\boxed{= 0}$$

4. Compute

$$\lim_{x\to\infty} \frac{2+e^x}{1-e^x}. \qquad \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{2+e^x}{1-e^x} = \lim_{x\to\infty} \frac{e^{-x}}{e^{-x}} \frac{2+e^x}{1-e^x}$$

$$= \lim_{x\to\infty} \frac{2e^{-x}+1}{e^{-x}-1} = \frac{2\cdot 0+1}{0-1}$$

$$= \frac{1}{-1} = \boxed{-1}$$

5. Compute
$$\lim_{x\to-\infty} \frac{2+e^x}{1-e^x}.$$

$$\lim_{x\to-\infty} \frac{2+e^x}{1-e^x} = \frac{2+0}{1-0} = 2$$

since $\lim_{x\to-\infty} e^x = 0.$

6. Compute
$$\lim_{x\to\infty} \ln(3+x) - \ln(1+x)$$

$$\lim_{x\to\infty} \ln(3+x) - \ln(1+x) = \lim_{x\to\infty} \ln\left(\frac{3+x}{1+x}\right)$$

$$\frac{\infty}{\infty}$$

$$= \ln\left(\lim_{x\to\infty} \frac{3+x}{1+x}\right)$$

$$= \ln\left(\lim_{x\to\infty} \frac{\frac{1}{x}}{\frac{1}{x}} \frac{3+x}{1+x}\right)$$

$$= \ln\left(\lim_{x\to\infty} \frac{3/x+1}{1/x+1}\right)$$

$$= \ln\left(\frac{0+1}{0+1}\right) = \ln(1)$$

$$\boxed{= 0}$$

7. Compute
$$\lim_{x\to\infty} \arctan(2^{-x})$$

Since $\lim_{x\to\infty} 2^{-x} = 0$, and since arctan is continuous,

$$\lim_{x\to\infty} \arctan(2^{-x}) = \arctan\left(\lim_{x\to\infty} 2^{-x}\right) = \arctan(0)$$
$$= 0$$

8. Compute
$$\lim_{x\to\infty} \frac{x^3 - 12x + 1}{x^4 + 7}$$

$$\lim_{x\to\infty} \frac{x^3-12x+1}{x^4+7} = \lim_{x\to\infty} \frac{x^{-1} - 12x^{-3} + x^{-4}}{1 + 7x^{-4}}$$

$$= \frac{0}{1+0} = \boxed{0}$$

9. Compute
$$\lim_{x\to-\infty} \frac{x^4 + 7}{x^3 - 12x + 1}$$

$$\lim_{x\to-\infty} \frac{x^4+7}{x^3-12x+1} = \lim_{x\to-\infty} x \cdot \left(\frac{x^3+7}{x^3-12x+1}\right)$$

$$= \lim_{x\to-\infty} x \left(\frac{1+7/x^3}{1-12/x^2+1/x^3}\right)$$

$$= -\infty \cdot 1 = \boxed{-\infty}$$