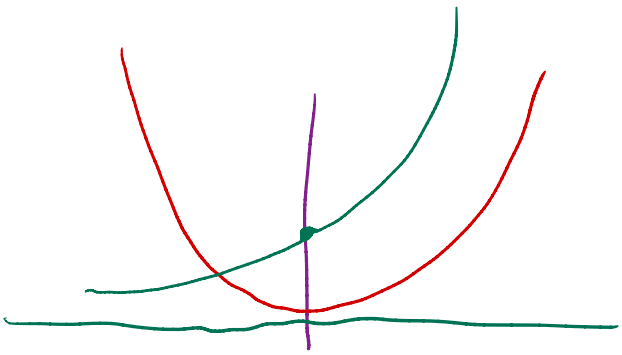
1. Continuous at $x = a$: $\lim_{x \to a} f(x) = f(a)$
2. A continuous function is continuous at each point in its domain.
3. If left and right limits disagree, then a limit does not exist.
4. Intermediate Value Theorem (one version): If a **continuous** function on $[a, b]$ is positive at a and negative at b, then it is zero somewhere in the middle.

1. Show that there is a number x such that

$$10^x = x^2.$$



Want x with $\underbrace{10^x - x^2}_{f(x)} = 0$

$f(0) = 10^0 - 0^2 = 1$

$f(-1) = 10^{-1} - (-1)^2 = -0.9$

Since $f(x)$ is continuous the IVT implies there is an x in $[-1, 0]$ with $f(x) = 10^x - x^2$.

2. True or false: taxi fare is a continuous function of distance traveled. Justify your answer. You may assume this generous taxi does not charge for waiting time.

The function is discontinuous.

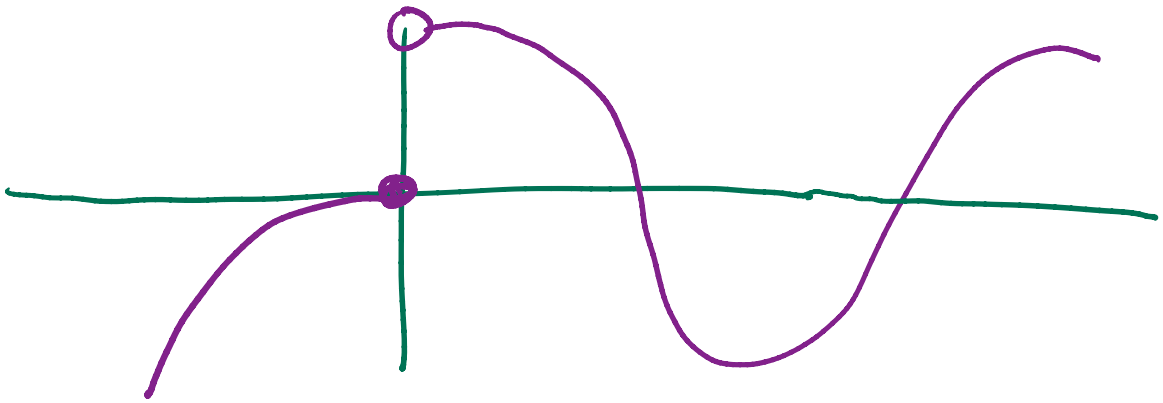
Each time the fare goes up, it does so by a jump.

3. Consider the function

$$f(x) = \begin{cases} \cos(x) & x > 0 \\ -x^2 & x \le 0 \end{cases}$$

$\lim_{x\to 0} f(x) = f(0)$ → 0

a) Sketch $f(x)$.



b) Compute $\lim_{x\to 0^+} f(x)$.

$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \cos(x) = \cos(0) = 1$

c) Compute $\lim_{x\to 0^-} f(x)$.

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} -x^2 = -0^2 = 0$

d) Is $f(x)$ continuous at $x = 0$? Justify your answer.

Since $\lim_{x\to 0^+} f(x) \neq \lim_{x\to 0^-} f(x)$, $\lim_{x\to 0} f(x)$ does not exist.

Continuity needs $\lim_{x\to 0} f(x) = f(0)$ but $\lim_{x\to 0} f(x)$ does not exist.

4. Consider the function

$$f(x) = \frac{\tan(3x)}{x}$$

a) What is the value of $f(0)$?

Not allowed

b) Using a calculator, estimate $\lim_{x\to 0} \tan(3x)/x$. Be sure to put your calculator in radians mode!

Let $f(x) = \tan(3x)/x$

x	$f(x)$
0.1	3.09...
0.01	3.0009...
0.001	3.000009...

Looks like $\lim_{x\to 0} f(x) = 3$

c) For what value of a is

$$g(x) = \begin{cases} \tan(3x)/x & x \neq 0 \\ a & x = 0 \end{cases}$$

continuous at $x = 0$?

Need $\lim_{x\to 0} g(x) = g(0)$. Since $g(0) = a$,

need $a = \lim_{x\to 0} g(x) = \lim_{x\to 0} \frac{\tan(3x)}{x} = 3$. So:

$a = 3$