For each limit in problems 1 through 5, verify that the expression is of the form 0/0 at the limit point. Then compute the limit using the "Limits don't care about one point" rule. For each limit computation, start by writing out the expression

$$\lim_{x \to a} f(x) =$$

for the specific values of f, a and x. Then carry on from here. Circle the equality in your computation where the "Limits don't care about one point" rule gets used.

1. Compute $\lim_{h \to 0} \frac{(3+h)^2 - 9}{h}$.

$$\lim_{h \to 0} \frac{(3+h)^2 - 9}{h} = \lim_{h \to 0} \frac{9 + 6h + h^2 - 9}{h}$$

$$= \lim_{h \to 0} \frac{6h + h^2}{h}$$

limits don't care

$$= \lim_{h \to 0} 6 + h = 6 + 0 = \boxed{6}$$

direct substitution

2. Compute $\lim_{x \to 3} \frac{\frac{1}{x} - \frac{1}{3}}{x - 3}$.

$$\lim_{x \to 3} \frac{\frac{1}{x} - \frac{1}{3}}{x - 3} = \lim_{x \to 3} \frac{\frac{3-x}{3x}}{x-3}$$

$$= \lim_{x \to 3} \frac{1}{x-3} \cdot \frac{3-x}{3x}$$

limits don't care

$$= \lim_{x \to 3} -\frac{1}{3x} = \frac{-1}{3 \cdot 3} = -\frac{1}{9}$$

DSP

3. Compute $\lim_{h\to 0} \frac{\sqrt{2+h}-\sqrt{2}}{h}$.

$$\lim_{h\to 0} \frac{\sqrt{2+h}-\sqrt{2}}{h} = \lim_{h\to 0} \frac{\sqrt{2+h}-\sqrt{2}}{h} \frac{\sqrt{2+h}+\sqrt{2}}{\sqrt{2+h}+\sqrt{2}}$$

limits don't care

$$= \lim_{h\to 0} \frac{h}{h(\sqrt{2+h}+\sqrt{2})}$$

$$= \lim_{h\to 0} \frac{1}{\sqrt{2+h}+\sqrt{2}}$$

DSP

$$= \frac{1}{\sqrt{2+0}+\sqrt{2}} = \boxed{\frac{1}{2\sqrt{2}}}$$

4. Compute $\lim_{h\to 0} \frac{\frac{1}{2+h}-\frac{1}{2}}{h}$.

$$\lim_{h\to 0} \frac{\frac{1}{2+h}-\frac{1}{2}}{h} = \lim_{h\to 0} \frac{\frac{2-(2+h)}{2(2+h)}}{h}$$

limits don't care

$$= \lim_{h\to 0} \frac{1}{h} \frac{h}{2(2+h)}$$

$$= \lim_{h\to 0} \frac{1}{2(2+h)} = \frac{1}{2(2+0)} = \boxed{\frac{1}{4}}$$

DSP

5. Compute $\lim_{x\to 2} \frac{x^3-8}{x-2}$.

$$\lim_{x\to 2} \frac{x^3-8}{x-2} = \lim_{x\to 2} \frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{x-2}}$$

LDC

$$= \lim_{x\to 2} x^2+2x+4$$

$$= 2^2+2\cdot 2+4$$

$$= \boxed{12}$$

6. Compute $\lim_{x\to 0} x^2 \sin(1/x)$. [Ask me about the Squeeze Theorem!]

STAY TUNED!

7. Compute $\lim_{x\to 6^+} \frac{6+|x|}{6-x}$.

Near $x=6$, $|x|=x$

$$\lim_{x\to 6^+} \frac{6+|x|}{6-x} = \lim_{x\to 6^+} \frac{6+x}{6-x} = \frac{12}{0^-} = \boxed{-\infty}$$

8. Compute $\lim_{x\to 6^-} \frac{6+|x|}{6-x}$.

Similarly,

$$\lim_{x\to 6^-} \frac{6+|x|}{6-x} = \lim_{x\to 6^-} \frac{6+x}{6-x} = \frac{12}{0^+} = \boxed{+\infty}$$