1. Limits are important to cope with 0/0.
2. We see 0/0 when computing instantaneous rates of change.
3. To estimate $\lim_{x\to a} f(x)$ you can substitute values of x close to a into f.
4. We also use limits to investigate 1/0 expressions, and when the limit exists the answer is typically $\pm\infty$.
5. 1/0 expressions often have different one-sided limits, $+\infty$ on one side and $-\infty$ on the other.

1. Estimate

$$\lim_{h\to 0} \frac{\overbrace{\sqrt{2+h}-\sqrt{2}}^{f(h)}}{h}$$

to 5 decimal digits.

h	value
0.1	0.349241....
0.01	0.353113.....
0.001	0.353509......
0.0001	0.353548....
0.00001	0.3535529
0.000001	$\boxed{0.35355}33$

$$\boxed{\lim_{h\to 0} f(h) \approx 0,35355}$$

2. Estimate

$$\lim_{x\to 0} \frac{x^2}{\cos(x)-1}$$

to 5 decimal digits.

$$f(x) = \frac{x^2}{\cos^2(x)-1}$$

x	f(x)
0.1	-2.0016....
0.01	-2.000016...
0.001	$-2.\underline{00000}0016.....$
0.0001	$-2.\underline{00000}000016.....$

$$\boxed{\lim_{x\to 2} f(x) \approx -2}$$

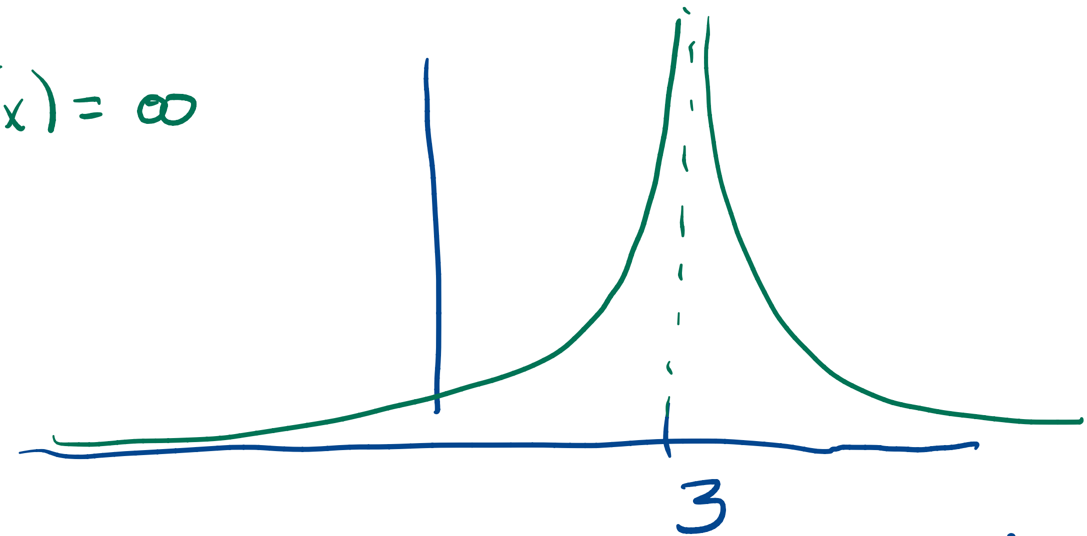
3. Sketch the graph of

$$f(x) = \frac{1}{(3-x)^2}.$$

Then determine

$$\lim_{x\to 3} f(x).$$

$$\lim_{x\to 3} f(x) = \infty$$



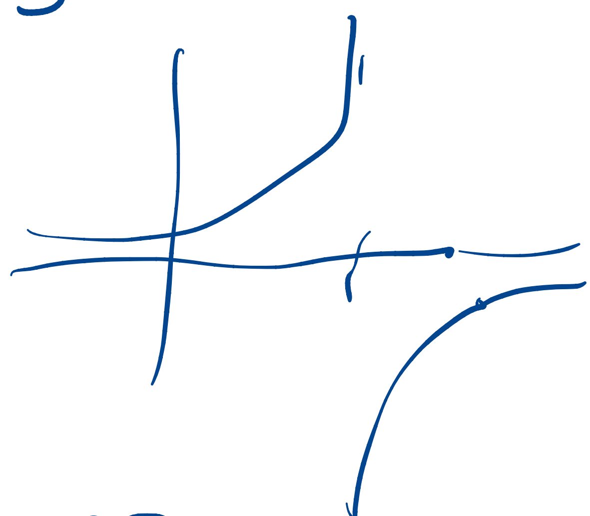
4. Determine

$$\lim_{x\to 3^+} \frac{1}{3-x}$$

and

$$\lim_{x\to 3^-} \frac{1}{3-x}.$$

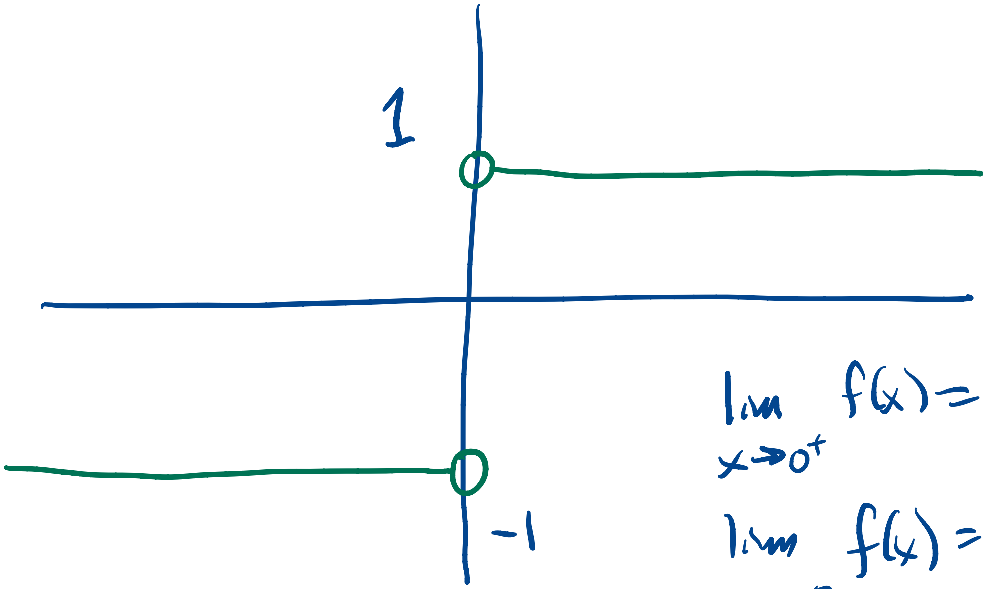
A sketch of the graph might be helpful.



$$\lim_{x\to 3^+} \frac{1}{3-x} = \frac{1}{0^-} = -\infty$$

$$\lim_{x\to 3^-} \frac{1}{3-x} = \frac{1}{0^+} = +\infty$$

5. Determine the left- and right-hand limits at 0 of $f(x) = x/|x|$.



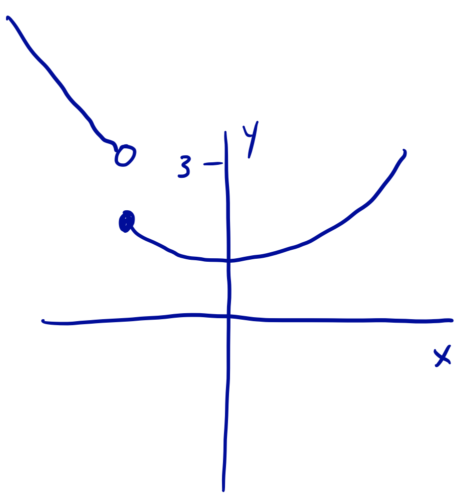
$\lim_{x\to 0^+} f(x) = 1$

$\lim_{x\to 0^-} f(x) = -1$

6. Suppose

$$g(x) = \begin{cases} x^2+1 & x \geq -1 \\ 2-x & x < -1. \end{cases}$$

Sketch the graph. Then determine if $\lim_{x\to -1} g(x)$ exists. If not, determine if the left- and right-hand limits exist.



$\lim_{x\to -1^-} g(x) = \lim_{x\to -1^-} 2-x = 3$

$\lim_{x\to -1^+} g(x) = \lim_{x\to -1^+} x^2+1 = 1+1 = 2$

7. Determine exactly
$$\lim_{x\to 2}\frac{x^2-7x+10}{x-2}$$

$$\lim_{x\to 2}\frac{x^2-7x+10}{x-2} = \lim_{x\to 2}\frac{(x-5)(x-2)}{x-2}$$

$$= \lim_{x\to 2} x-5 = -3.$$

8. Determine
$$\lim_{x\to 0^+} 10^{-\frac{1}{x}}$$
and
$$\lim_{x\to 0^-} 10^{-\frac{1}{x}}.$$

As $x\to 0^+$, $-\frac{1}{x} \to \frac{-1}{0^+} = -\infty$ and $10^{-\frac{1}{x}} \to 0$.

As $x\to 0^-$ $-\frac{1}{x} \to \frac{-1}{0^-} = +\infty$ and $10^{-1/x} \to \infty$.