A car is traveling down the road, and we have measured the distance it traveled at various times over a period of one hour. In this worksheet we will discuss how to convert distance measurements into speed measurements for the car.

To begin, you will need to access a Google Docs spreadsheet that contains the car measurements. To do this:

1. Go to the Math 251 website:
   `https://uaf-math251.github.io`.

2. Vist the link for section FXB resources.

3. Follow the instructions under Section 2.1 Worksheet.

Now answer the following questions.

1. **How far did the car travel during the hour?**

   36.45 miles

2. **If you had to give just one answer, how fast would you say the car was traveling during the hour? Express your answer first in units of miles per hour (because that will feel most comfortable), but then also in miles per minute (because these units will be used for the rest of the worksheet).**

   36.45 miles/hour $= 0.61$ miles/minute

3. **If you had to give just one answer, how fast would you say the car was traveling during the first half hour? Express your answer first in units of miles per hour and then also in miles per minute.** $\dfrac{12.99}{0.5} = 25.98 \frac{miles}{hour}$ $\dfrac{12.99}{30} = 0.43$ miles/minute

4. **The speeds you just computed are called *average speeds*. Now compute the average speed over the the second half hour. Express your answer in units of miles per minute. You can use the spreadsheet to do these computations for you, if you would like. If you have questions on how to do this, ask!** $\dfrac{36.45 - 12.99}{30} = 0.782$ miles/minute

5. **Your computations should show that the average speeds are not the same during the two half-hour periods. Which one was slower? How could you have predicted this from looking at the graph of distance traveled?**

   First is slower. Graph is not as steep.

6. **Just looking at the graph, decide if the car is traveling faster around the 20 minute mark, or around the 40 minute mark.**

7. **Now compute the average speeds (in miles per minute) over the time interval from 20 to 25 minutes, and the time interval from 40 to 45 minutes.**
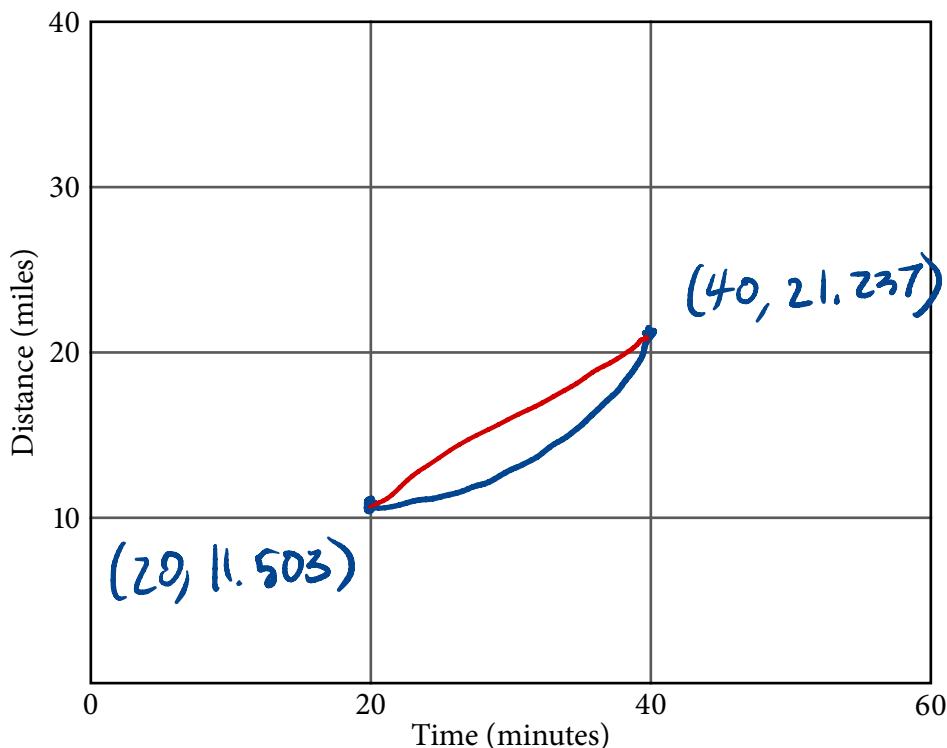
   $t: [20,25]$ $0.86 \frac{miles}{minute}$ $\Big|$ $t: [40,45]$ $1.105 \frac{miles}{minute}$

8. Here's a happy accident of geometry. We won't do much with it on this worksheet, but we'll be talking about it very soon in lecture. In the graph below, do the following:
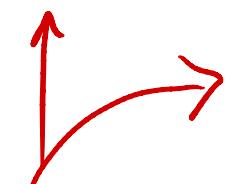
    a) Plot the two points corresponding to distance traveled at $t = 20$ and $t = 40$ minutes.

    b) Draw the straight line connecting these two points. This line will be important.

    c) Make a rough sketch of the distance traveled curve over the interval from $t = 20$ to $t = 40$ by copying the graph you see in the spreadsheet. This curve is just for context, it doesn't have to be super accurate.

    Then answer the following:

    i) What is the slope of the line you sketched in part (b) above?

    ii) What is the average speed of the car over the time interval $t = 20$ to $t = 40$?

SAME!

slope: $\dfrac{21.237 - 11.503}{20} = 0.509$ miles/minute

average speed: $\dfrac{21.237 - 11.503}{20} = 0.509$ miles/minute

9. Although the spreadsheet only shows, right now, the distance traveled at 5 minute intervals, you can determine the distance at other times, too. For example, to compute the distance traveled after 3 minutes and 45 seconds, go to a blank cell and enter =DIST(3.75). After a pause, you should see that the distance traveled at that point is about 3.67 miles. Now use your newly-found power to compute the distance traveled after 41 minutes and after 42 minutes, and the average speed over the time interval from 41 to 42 minutes.

$$\frac{change \text{ in } distance}{change \text{ in } time} \longrightarrow \frac{1.11 \text{ miles}}{1 \text{ minute}}$$

10. Let's use the notation $d(t)$ for the distance traveled, so $d(40) \approx 21.24$, for example. We want to write down a formula for the average speed over the time interval $[t_0, t_1]$.

(a) Using only the symbols d, t_0, t_1 and arithmetic operations, write an expression for the total distance traveled over the time interval $[t_0, t_1]$.

(b) What is the length of the time interval $[t_0, t_1]$?

(c) Write down a formula for the average speed over the time interval $[t_0, t_1]$.

a) $d(t_1) - d(t_0) \longrightarrow \Delta \text{ distance}$

b) $t_1 - t_0 \longrightarrow \Delta \text{ time}$

c) $\dfrac{d(t_1) - d(t_0)}{t_1 - t_0}$ $\dfrac{\Delta \text{ dist}}{\Delta \text{ time}}$

11. Sometimes it will be handy to use time intervals of the form $[t_0, t_0 + h]$, so t_0 is the start of the interval and h is the length of the interval. Substitute $t_1 = t_0 + h$ into your expression for average speed in your previous problem and simplify.

$$\frac{d(t_0 + h) - d(t_0)}{(t_0 + h) - t_0} = \frac{d(t_0 + h) - d(t_0)}{h}$$

12. Now let's contemplate the speed right at time 41 minutes. This corresponds to a time interval with $t_0 = 41$ and $t_1 = 41$ (or alternatively with $t_0 = 41$ and $h = 0$). Explain why your formulas for average speed from the previous two problems are not immediately helpful now.

$$\Delta t = 0 \text{ minutes}$$
$$\Delta d = 0 \text{ miles} \qquad \frac{0}{0} \longrightarrow \text{undefined!}$$

13. Instead, we can work with average speeds over short time intervals near time $t = 41$ minutes. Use the spreadsheet to compute the average speeds over the time intervals $[41, 41+h]$ for:

(a) $h = 1$ minutes $1.11341466\ 4$ miles/minute

(b) $h = 0.1$ minutes $1.10898178\ 8$ miles/minute

(c) $h = 0.01$ minutes $1.10837336\ 4$ miles/minute

(d) $h = 0.001$ minutes $1.10831087\ 7$ miles/minute

(e) $h = 0.0001$ minutes $1.10830461\ 7$ miles/minute

(f) $h = 0.00001$ minutes $1.10830398\ 6$ miles/minute

To answer this, you might find it helpful to make a little table in the spreadsheet with two columns, one with h and one with average speed for that value of h.

14. Write down an estimate for the for the speed of the car at time $t = 41$ minutes to as many digits that you feel confident in, given your answers to the previous problem. Estimate the error in your approximation, and express that error in miles per minute, miles per hour, and inches per hour. You may find it helpful to know that there are 63360 inches in a mile.

$$1.10830\ 3986$$
$$\longrightarrow \text{trust}$$

$$\text{error} \approx 10^{-5} \ \frac{\text{miles}}{\text{minute}} \approx 38 \text{ inches}/\text{hour}$$