1. You start with a 100g lump of a radioactive isotope. A year later the lump has a mass of 97.7g. What is the half life of the isotope?

See next page.

2. At time $t = 0$ minutes, a colony of E. coli has 10000 cells. The population is growing exponentially, and after 60 minutes it has 90000 members. Find a function of the form

$$p(t) = C\,10^{at}$$

that describes the population size.

See two pages later.

3. The function $f(x) = 2^{-3x}$ can be written in the form $f(x) = 10^{-ax}$ for a certain constant a. Determine the value of a.

$$2^{-3x} = 10^{-ax}$$

$$-3x\ln(2) = -ax\ln(10)$$

$$\boxed{a = \frac{3\ln(2)}{\ln(10)}}$$

1) $m(t) = C \cdot \left(\frac{1}{2}\right)^{at}$, half-life: $1/a$

$m(0) = 100$; $m(0) = C \cdot 1 = C \Rightarrow C = 100$

$m(1) = 97.7$; $m(1) = 100 \left(\frac{1}{2}\right)^{a}$

$$\Rightarrow \quad 100 \left(\frac{1}{2}\right)^{a} = 97.7$$

$$\Rightarrow \quad \left(1/2\right)^{a} = 0.977$$

$$\Rightarrow \quad a \log_{10}\left(\frac{1}{2}\right) = 0.977$$

$$\Rightarrow \quad \boxed{\frac{1}{a} = \frac{\log_{10}(1/2)}{0.977} \approx 29.78 \text{ years}}$$

2) $P(t) = C \cdot 10^{at}$

$$\left.\begin{aligned} P(0) &= 10000 \\ P(60) &= 90000 \end{aligned}\right] \text{given}$$

but $P(0) = C \cdot 10^{a \cdot 0} = C \Rightarrow \boxed{C = 10000}$

Also, $P(60) = 10000\ 10^{a \cdot 60}$

So

$$10000\ 10^{a60} = 90000$$

$$10^{a \cdot 60} = 9$$

$$a \cdot 60 = \log_{10} 9$$

$$\boxed{a = \frac{\log_{10} 9}{60} \approx 0.0159}$$

4. Use the change of base formula to rewrite $\log_{10}(7)$ in terms of the natural logarithm, ln.

$$\log_{10} 7 = \frac{\ln(7)}{\ln(10)}$$

5. Solve the following equation for x:

$$\ln(x) + \ln(x-1) = 2.$$

$$\ln(x(x-1)) = 2$$

$$x(x-1) = e^2$$

$$x^2 - x - e^2 = 0$$

$$x = \frac{1 \pm \sqrt{1+4e^2}}{2}$$

can only keep $\frac{1+\sqrt{1+4e^2}}{2}$

since the other is negative and illegal for

6. Find the inverse function of $f(x) = 1 + \sqrt{2-3x}$. Remember:

a) Write $y = f(x)$.

b) Solve for x.

c) The resulting expression in terms of y is $f^{-1}(y)$.

$$y = 1 + \sqrt{2-3x}$$

$$y - 1 = \sqrt{2-3x}$$

$$(y-1)^2 = 2-3x$$

$$3x = 2 - (y-1)^2$$

$$x = \frac{2-(y-1)^2}{3}$$

$$f^{-1}(x) = \frac{2-(y-1)^2}{3}$$