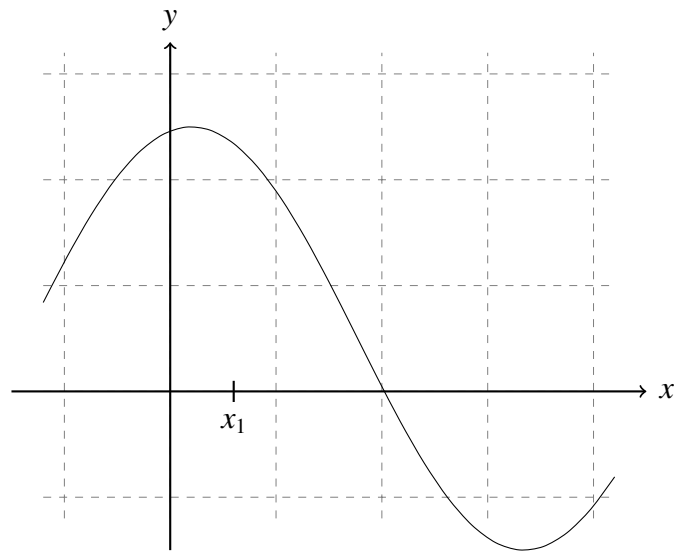


11. (10 points)

a. A generic graph $y = f(x)$ is shown and a first approximation x_1 is indicated. Show, by adding to the sketch, how Newton's method would find the next approximation x_2 .



b. For the equation $x^3 - 4x + 2 = 0$ and the value $x_1 = -2$, compute x_2 from Newton's method.

12. (Extra Credit: 5 points)

Find **and simplify** the derivative of the function:

$$h(x) = \int_1^{e^x} \ln t \, dt$$

Explain your steps.

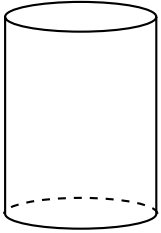
$$G(x) = \int_1^x \ln(t) \, dt$$

$$10 \quad G(e^x)$$

$$\frac{d}{dx} G(e^x)$$

4. (10 points)

The height of a right circular cylinder is increasing at rate of 3 meters per second while its volume remains constant. (See figure below.) At what rate is the radius changing when the radius and height are both 10 meters?



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1. (10 points)

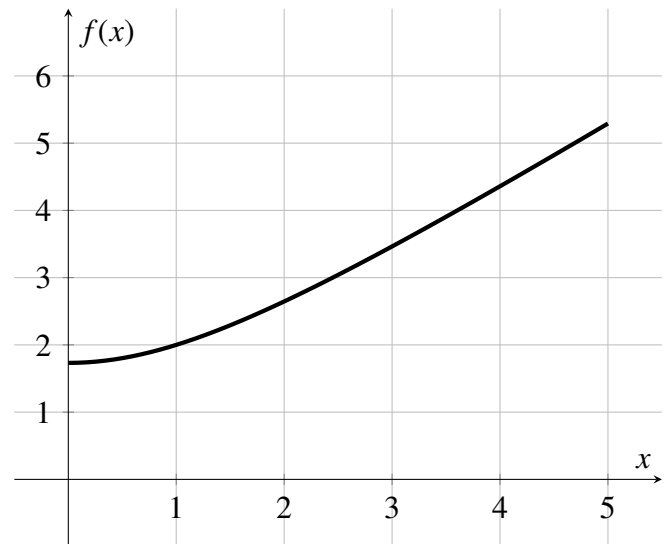
Find an equation of the tangent line to the curve at $x = e$: $y = x^2 \ln x$

2. (10 points)

The graph of the function $f(x) = \sqrt{x^2 + 3}$ is shown.

a. On the graph sketch 3 rectangles, using left endpoints, that would be used to approximate

$$\int_1^4 \sqrt{x^2 + 3} dx.$$



b. Compute the approximation in part (a). You do not need to simplify, but your answer should be in a form where a calculator would compute a numerical value.

1. (10 points)

Sketch a graph $H(x)$ with all of the properties below. Label your graph.

- The domain of $H(x)$ is $(-\infty, 3) \cup (3, \infty)$.
- $H(0) = 1$
- $\lim_{x \rightarrow 0^-} H(x) = 2$
- $\lim_{x \rightarrow 0^+} H(x) = 0$
- $\lim_{x \rightarrow 3} H(x) = \infty$
- $H'(x) < 0$ and $H''(x) < 0$ on the interval $(-\infty, 0)$
- H has an inflection point when $x = 5$

5. (10 points)

Find any horizontal or vertical asymptotes for the function $f(x) = \frac{2x^2-3x}{5x^2-10}$. Use limits to justify your answer(s). If no asymptote exists, explain why.

6. (10 points)

A homeowner wants to minimize the cost of heating a building over the next 10 years. Adding x inches of insulation in the attic costs \$100 per inch and results in heating costs of $1000/(2+x)$ dollars over 1 year. How many inches of insulation should be installed in order to minimize the total costs over a 10 year period? Justify your answer. (By **total costs**, we mean both the initial cost of insulating the building plus the annual heating costs.)

9. (10 points)

Short Answer

a. A population of chickadees is increasing at a rate of $r(t)$ chickadees per year. What does $\int_1^4 r(t) dt = 400$ mean? Make sure to include units in your answer.

b. Let $y = -3 + 5(x - 4)$ be an equation of the tangent line to the graph of $f(x)$ at $x = 4$. Is it possible to determine $f(4)$ or $f'(4)$? Explain your answer.

c. Let $C(T)$ be the number of chirps per second of a male cricket as a function of temperature, T , in degrees Fahrenheit. In the context of the problem, interpret $C'(70) = 2$. Make sure to include units in your answer.

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7. (15 points)

A particle moves so that its velocity (in m/sec) at time t sec is

$$v(t) = t^2 + 7.$$

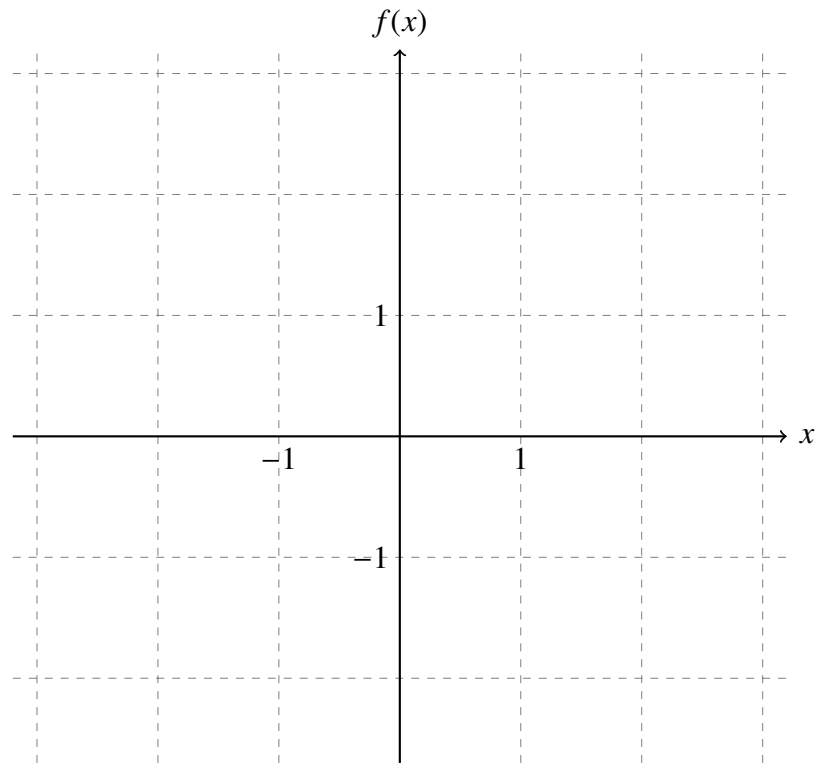
a. What is the average rate of change of the velocity from time $t = 2$ to $t = 3$? Simplify, and give units.

b. Using the limit definition of the derivative, compute $v'(2)$. (No credit will be given for using a different method to compute the derivative.)

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Problem 9 continued....

- e. Sketch the graph on the axes:



10. (10 points)

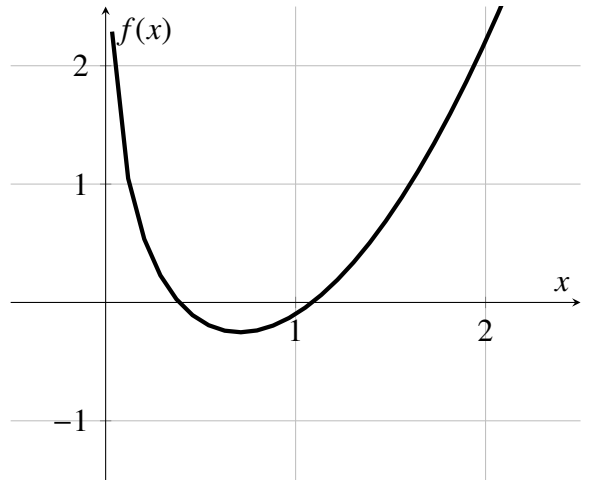
A farmer has 400 meters of fencing and wants to fence off a rectangular field that borders a straight river. No fencing is needed along the river, which forms one side of the rectangle. What are the dimensions of the field that has the largest area?

- a. Draw a sketch and choose labels for the sides.
- b. Solve the problem. Indicate units in your answer.

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11. (10 points)

The graph of the function $f(x) = x^2 - \ln(3x)$ is shown.



a. Suppose Newton's method is used to find an approximate solution to $f(x) = 0$ from an initial guess of $x_1 = 2$. Sketch on the graph how the next approximation x_2 will be found, labeling its location on the x -axis.

b. For $x_1 = 2$, give a formula for x_2 . You do not need to simplify, but your answer should be in a form where a calculator would compute a numerical value.

c. What value of x_1 might you use if you wanted to find the **smaller** solution of $f(x) = 0$?

Extra Credit. (3 points)

Compute the following integral by interpreting it as an area:

$$\int_0^4 \sqrt{4 - (x - 2)^2} dx$$