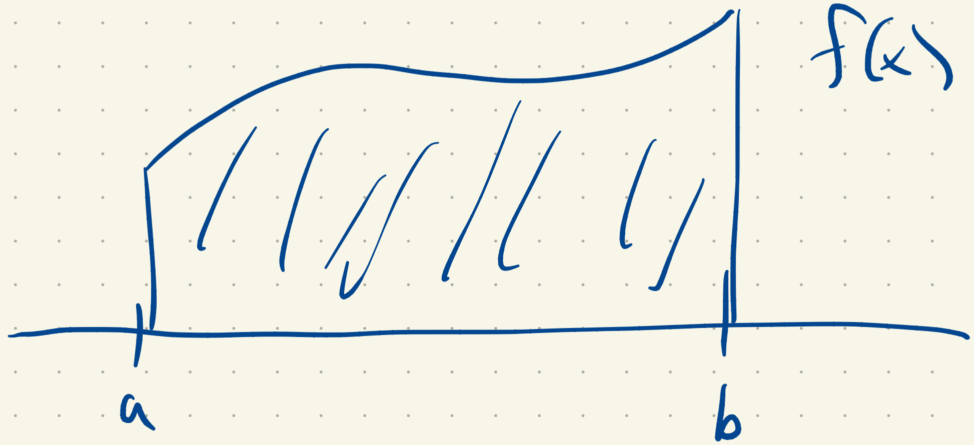
$$\int_a^b f(x)\,dx$$



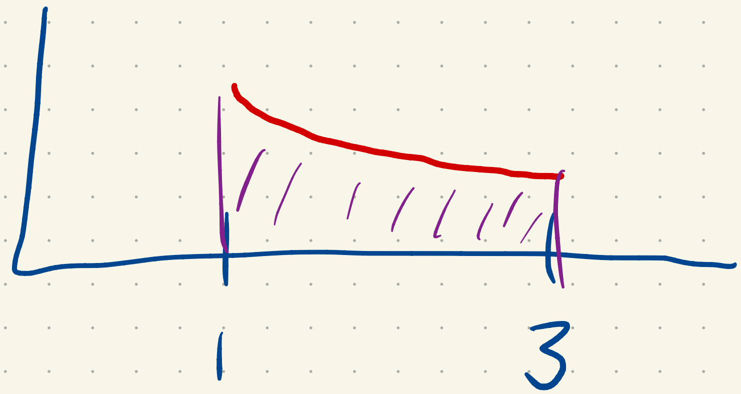
FTC II

1) Find an antiderivative $F(x)$ for $f(x)$.

(HARD)

2) $$\int_a^b f(x)\,dx = F(b) - F(a) = F(x)\Big|_a^b$$

$$\int_1^3 e^{-t}\,dt$$



$$F(t) = -e^{-t}$$

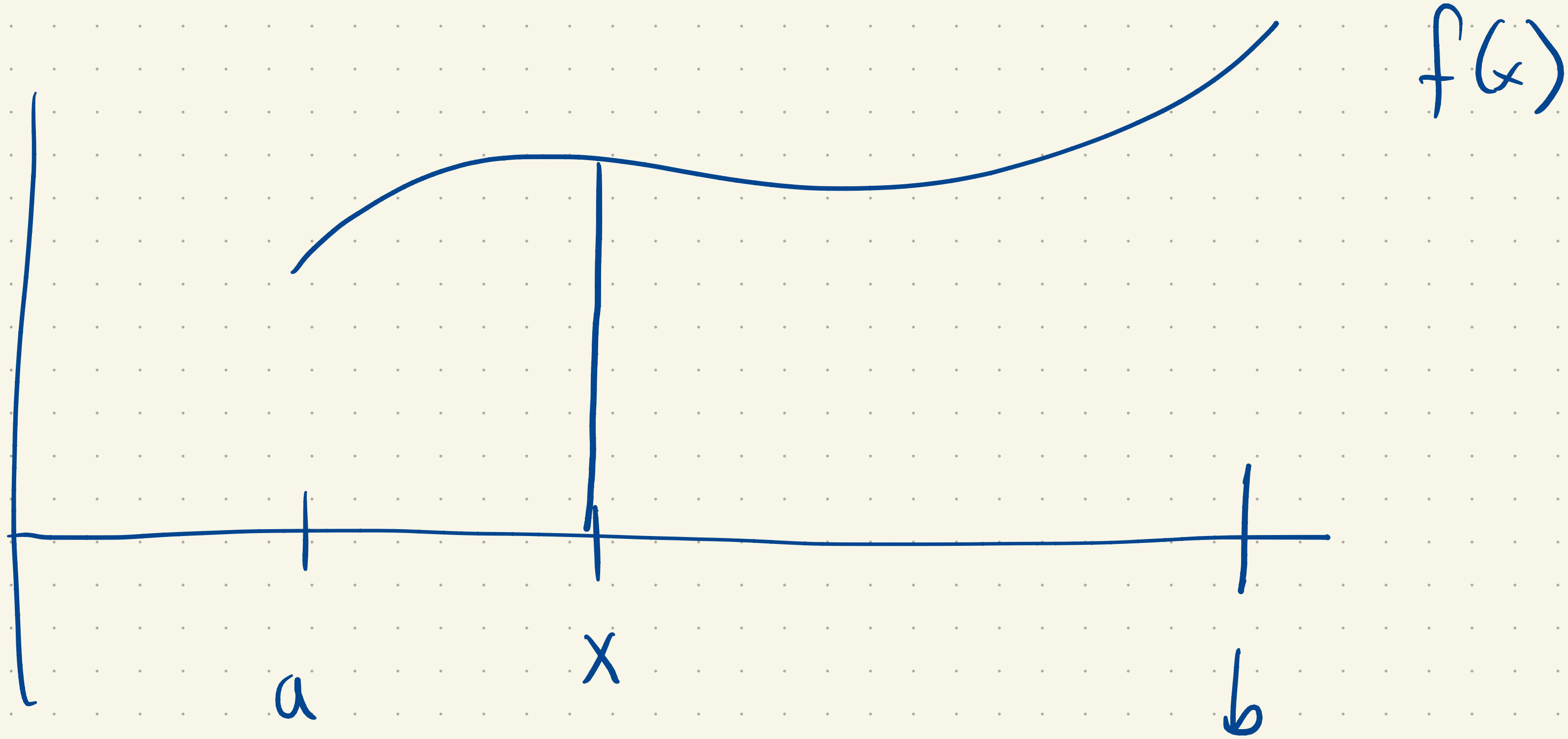
$$F'(t) = -e^{-t}\cdot(-1)$$

$$= e^{-t} \checkmark$$

$$\int_1^3 e^{-t}\,dt = -e^{-t}\Big|_1^3$$

$$= -e^{-3} - \left(-e^{-1}\right)$$

$$= e^{-1} - e^{-3} = 0.318\ldots$$

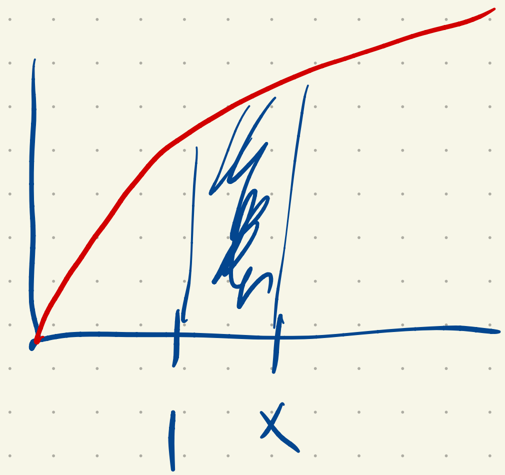


$$G(x) = \int_a^x f(s)\,ds$$

$$G(a) = 0$$

$$G(b) = \int_a^b f(s)\,ds$$

FTC I: $G'(x) = f(x)$ ($f(x)$ is continuous)



$$f(x) = \sqrt{x + x^3}$$

$$G(x) = \int_1^x \sqrt{t + t^3}\,dt$$

$$\frac{d}{dx} \int_1^x \sqrt{t + t^3}\,dt = \sqrt{x + x^3}$$

Sketch of proof:

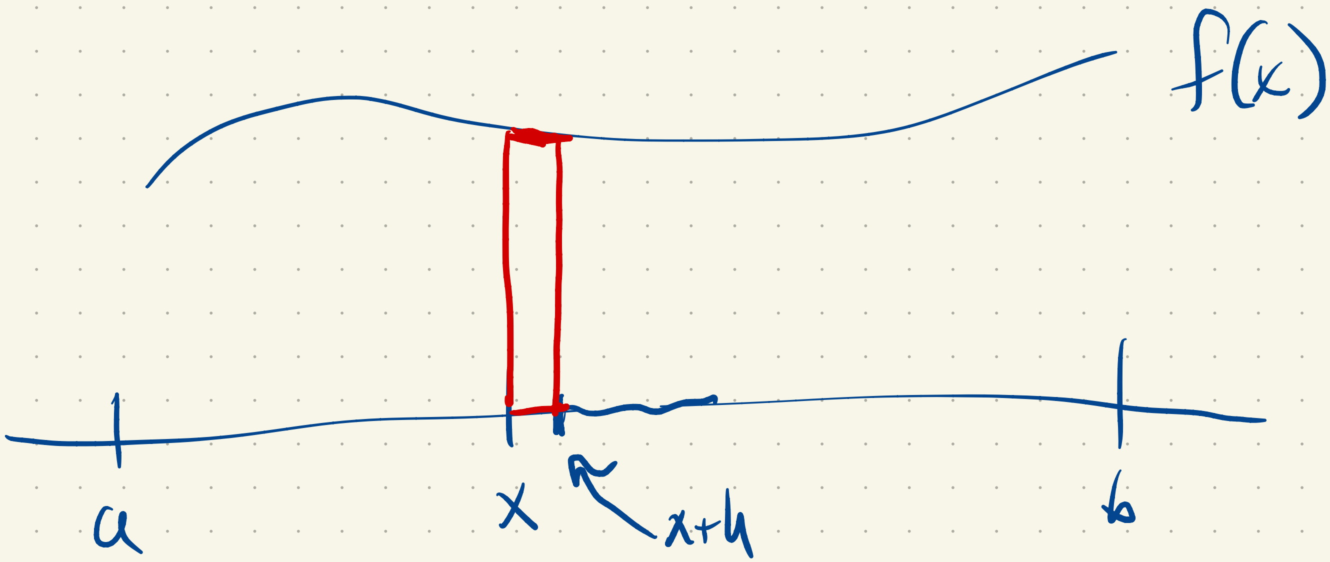
$$G(x) = \int_a^x f(s)\,ds$$

$$G'(x) = \lim_{h \to 0} \frac{G(x+h) - G(x)}{h}$$

$$\int_a^x f(s)\,ds + \int_x^{x+h} f(s)\,ds = \int_a^{x+h} f(s)\,ds$$

$$G(x) + \int_x^{x+h} f(s)\,ds = G(x+h)$$

$$G(x+h) - G(x) = \int_x^{x+h} f(s)\,ds$$



$$G'(x) = \lim_{h \to 0} \frac{1}{h} \underbrace{\int_x^{x+h} f(s)\,ds}$$

approximately area of a rectangle with width h and height $f(x)$

so with area

$f(x) \cdot h$

$= f(x)$

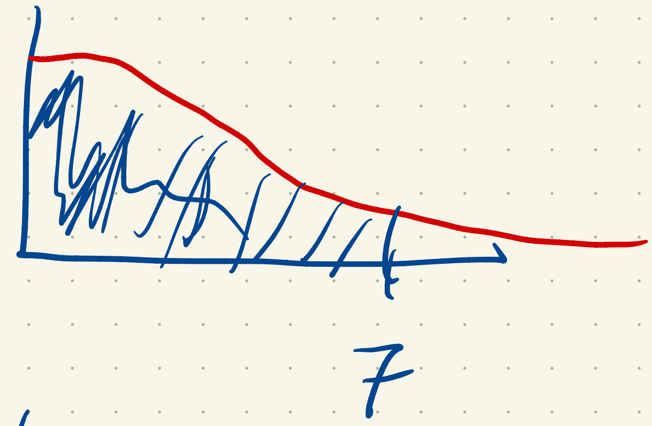
$$\frac{d}{dx} \int_3^x \cos(\theta)\, d\theta = \cos(x)$$

$G(x) = \int_a^x f(s)\, ds$

$G'(x) = f(x)$

$$\frac{d}{dx} \int_0^x e^{-t^2}\, dt = e^{-x^2}$$

$$\frac{d}{dx}\int_7^x e^{-t^2}\,dt = e^{-x^2}$$



$$\int_0^x e^{-t^2}\,dt = \int_0^7 e^{-t^2}\,dt + \int_7^x e^{-t^2}\,dt$$

$$\frac{d}{dx}\int_x^2 t^2\sin(t)\,dt = \frac{d}{dx}\left[-\int_2^x t^2\sin(t)\,dt\right]$$

$$= -\frac{d}{dx}\int_2^x t^2\sin(t)\,dt$$

$$= -x^2 \sin(x)$$

$$\frac{d}{dx} \boxed{\int_3^{x^2} \cos(2\theta)\, d\theta} \longrightarrow G(x^2)$$

$$G(x) = \int_3^{x} \cos(2\theta)\, d\theta$$

$$\frac{d}{dx} \cos(x^2)$$

$$\frac{d}{dx} G(x^2)$$

$$G'(x) = \cos(2x) \qquad G(x^2)$$

$$\frac{d}{dx} G(x^2) = G'(x^2) \cdot 2x$$

$$\frac{d}{dx} \int_3^{x^2} \cos(2\theta)\, d\theta = \cos(2x^2) \cdot 2x$$

$$\frac{d}{dx}\left[\int_{-x}^{2x} \sqrt{1+s^2}\, ds\right]$$

$$= \int_{-x}^{7} \sqrt{1+s^2}\, ds + \int_{7}^{2x} \sqrt{1+s^2}\, ds$$

$$= -\int_{7}^{-x} \sqrt{1+s^2}\, ds + \int_{7}^{2x} \sqrt{1+s^2}\, ds$$

$$\frac{d}{dx}\int_7^{2x}\sqrt{1+s^2}\,ds = \boxed{\sqrt{1+(2x)^2}\cdot 2}$$

$$G(x) = \int_7^{x}\sqrt{1+s^2}\,ds \rightarrow G'(x) = \sqrt{1+x^2}$$

$$\frac{d}{dx}G(2x) = G'(2x)\cdot 2$$

$$\frac{d}{dx} \int_7^{-x} \sqrt{1+s^2}\, ds = \sqrt{1+(-x)^2} \cdot (-1)$$

$$\frac{d}{dx} \int_{-x}^{2x} \sqrt{1+s^2}\, ds$$

$$= \sqrt{1+x^2} + 2\sqrt{1+(2x)^2}$$