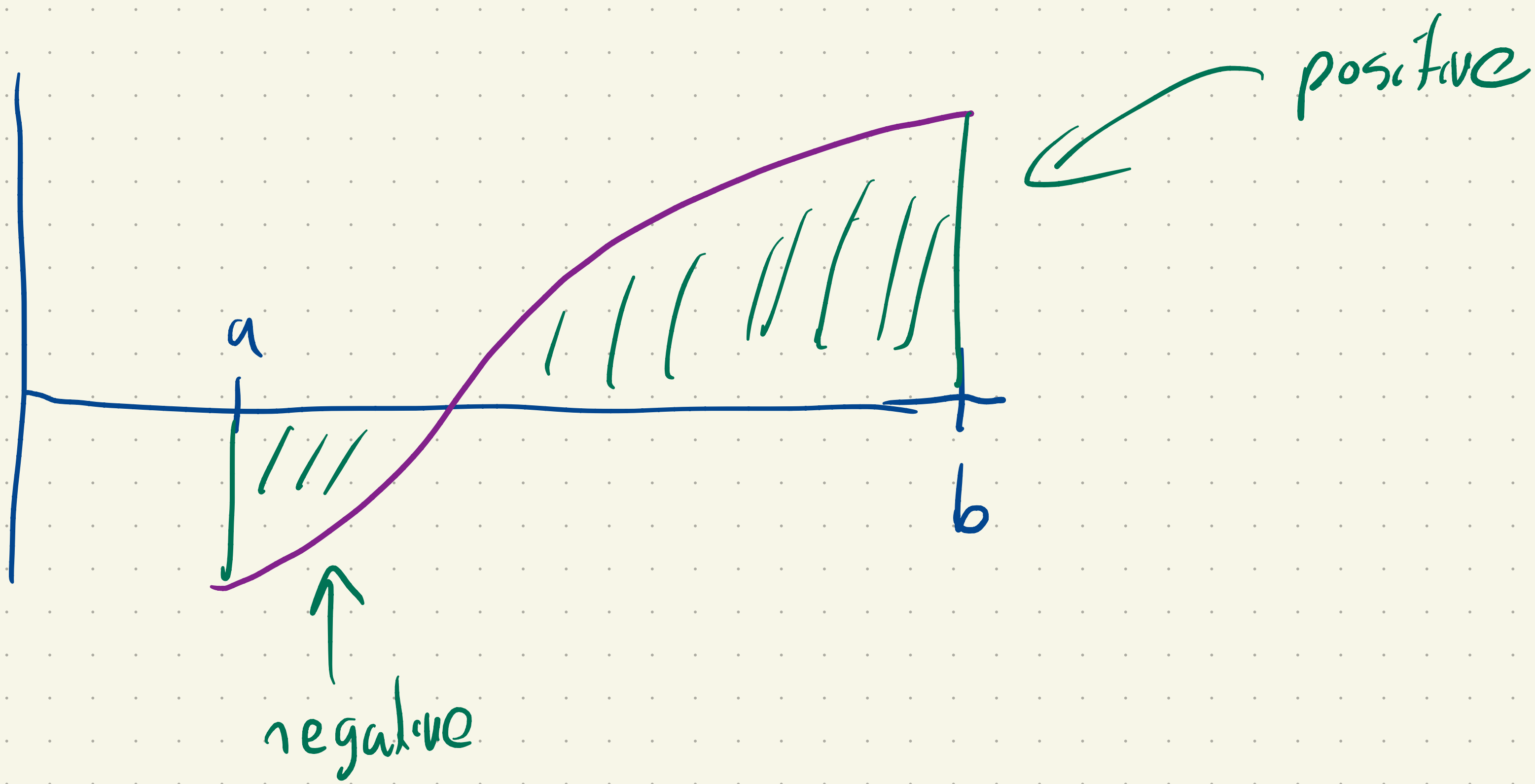
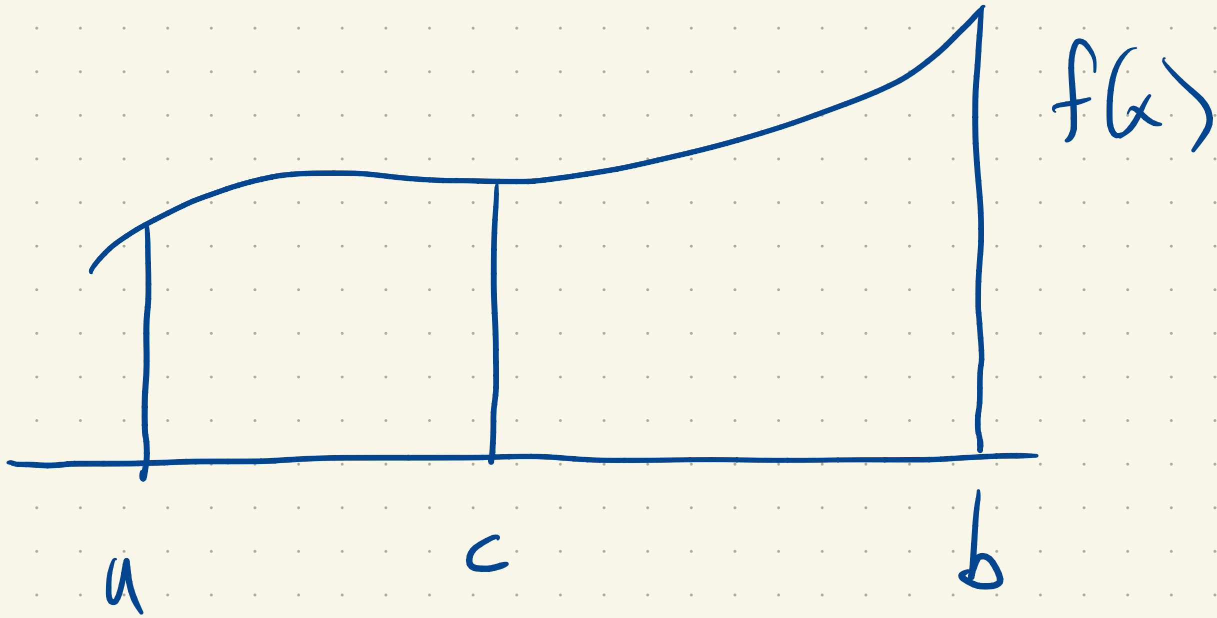
$$\boxed{\int_a^b f(x)\,dx} \longrightarrow \text{number}$$



$v(t)$ is velocity

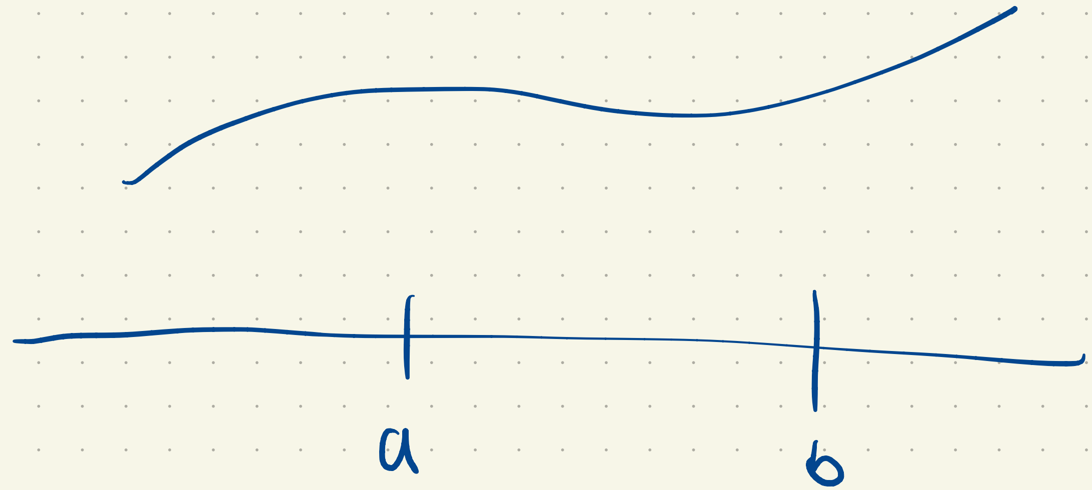
$$\int_a^b v(t)\,dt$$ net distance traveled.

4)



$$\int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \int_a^b f(x)\,dx$$

5)



$$\int_a^a f(x)\,dx = 0$$

$$\int_1^3 x^2\,dx$$

6)

$$\int_3^1 x^2\,dx$$

$$\int_a^b f(x)\,dx + \int_b^a f(x)\,dx = \int_a^a f(x)\,dx$$

$$= 0$$

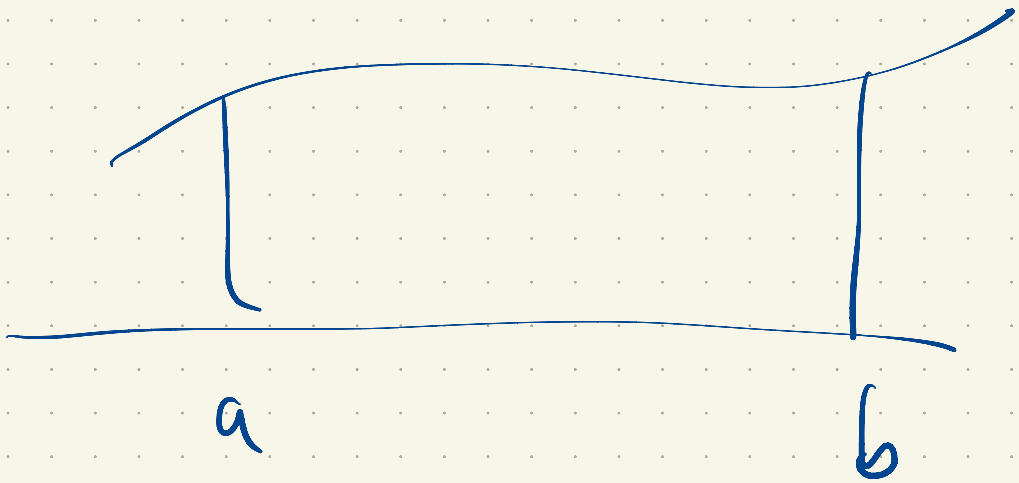
$$\Delta x = \frac{b-a}{n}$$

$$\boxed{\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx}$$

$$\sum_{k=1}^{n} f(x_k^*)\,\Delta x$$

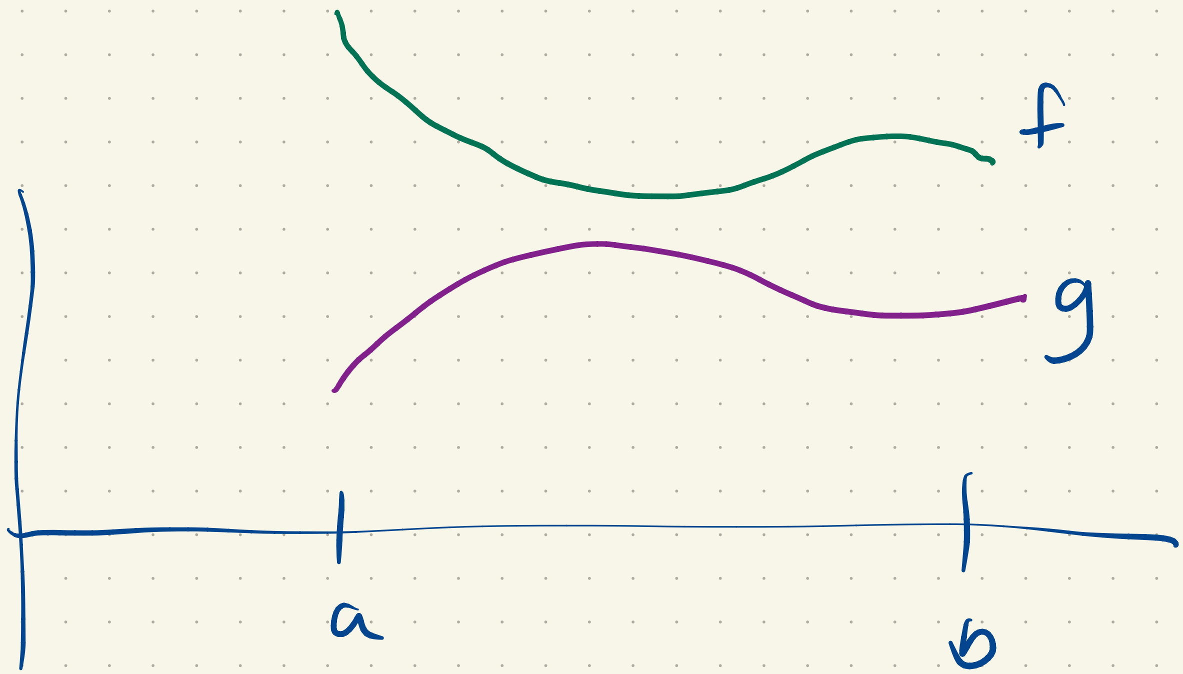
7) If $\underline{\underline{f(x)}} \geq 0$ on $[a, b]$

$$\int_a^b f(x)\,dx \geq 0$$



8) $f(x), g(x)$ $f(x) \geq g(x)$ on $[a,b]$

$$\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$$



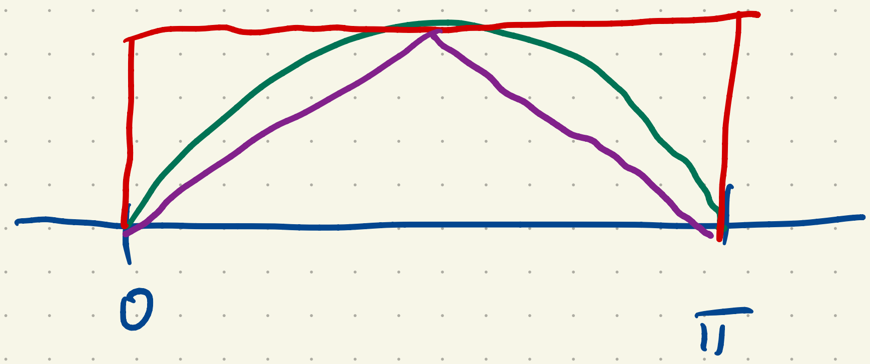
$$\int_a^b f(x) - g(x)\,dx \geq 0$$

$$f(x) \geq g(x)$$

$$f(x) - g(x) \geq 0$$

$$\int_a^b f(x)\,dx - \int_a^b g(x)\,dx \geq 0$$

$$\int_0^{\pi} \sin(x)\,dx = 2$$



$$1.5\ldots = \frac{\pi}{2} \leq \int_0^{\pi} \sin(x)\,dx \leq \pi = 3.14\ldots$$

$$F(x) = -\cos(x) \qquad F'(x) = \sin(x)$$

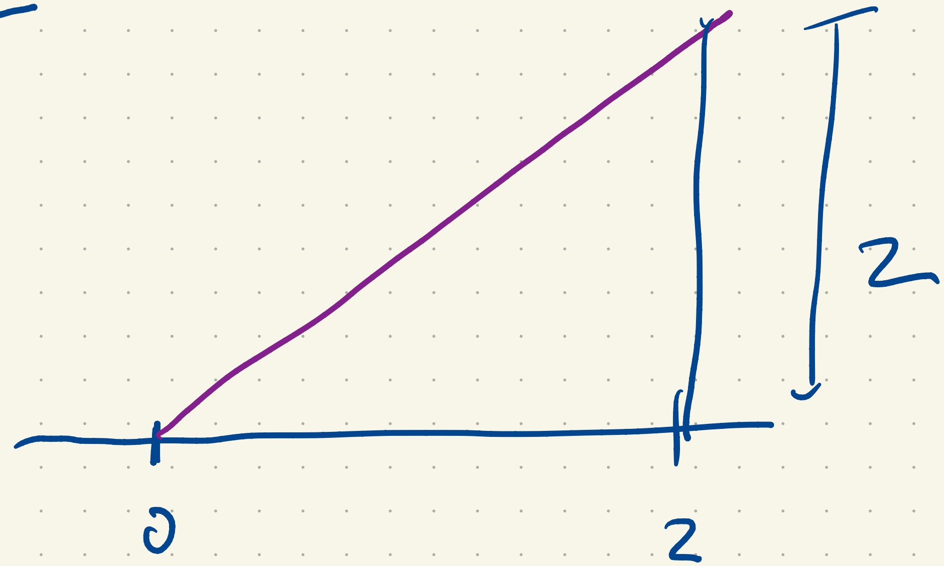
$$\int_0^{\pi} \sin(x)\,dx = F(\pi) - F(0)$$

$$= -\cos(\pi) - \left(-\cos(0)\right)$$

$$= -(-1) - (-1)$$

$$= 2$$

$$\int_0^2 x\,dx = 2$$



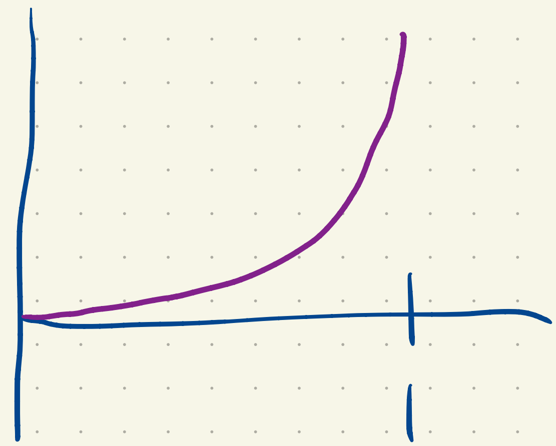
$$F(x) = \frac{1}{2}x^2$$

$$\int_0^2 x\,dx = F(2) - F(0)$$

$$= \frac{1}{2}2^2 - \frac{1}{2}0^2$$

$$= \frac{1}{2} \cdot 4 = 2 \quad \checkmark$$

$$\int_0^1 x^2\,dx$$

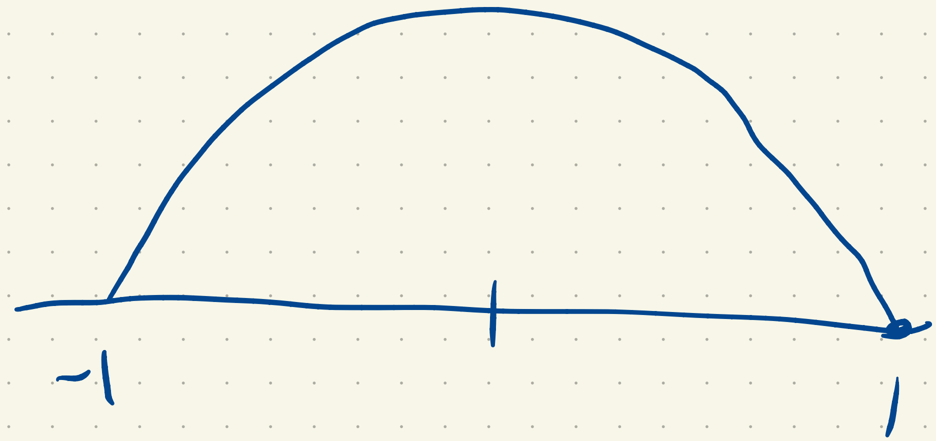


$$\int_0^1 x^2\,dx = \underbrace{F(1) - F(0)} = \frac{1}{3}$$

$$F(x) = \frac{1}{3}x^3$$

$$F(x)\Big|_0^1 = F(1) - F(0)$$

$$\int_{-1}^{1} 1 - x^2 \, dx$$



$$\int_{-1}^{1} 1 - x^2 \, dx = x - \frac{x^3}{3} \Bigg|_{-1}^{1} = \left(1 - \frac{1^3}{3}\right) - \left(-1 - \frac{(-1)^3}{3}\right)$$

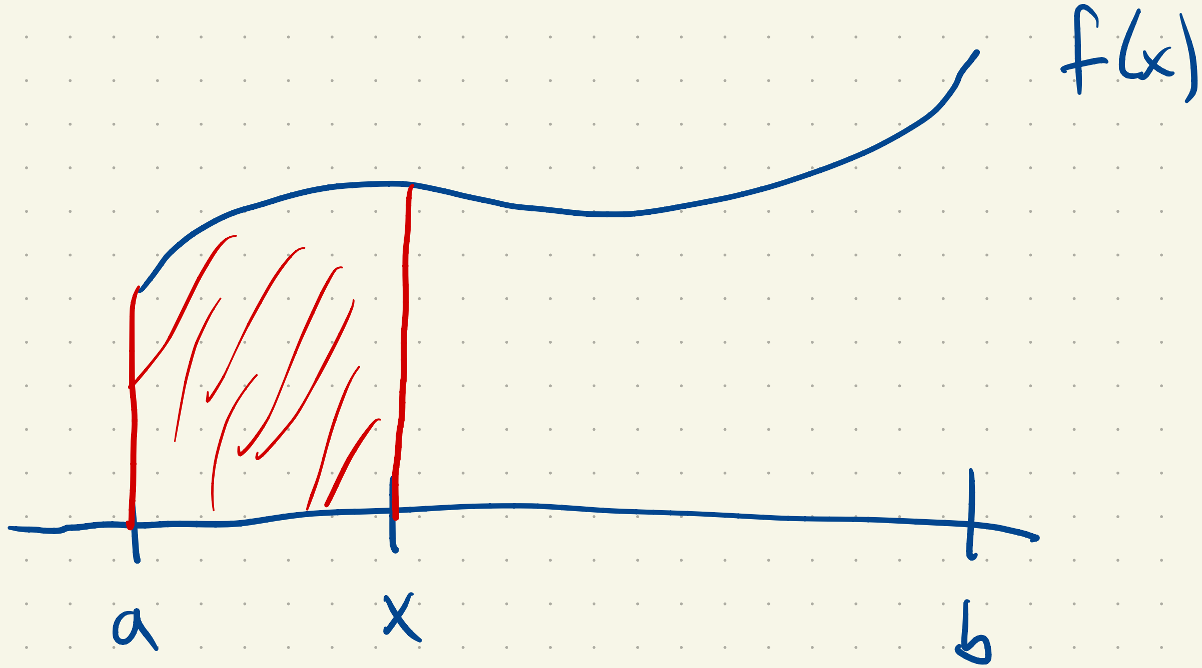
$$= 1 - \frac{1}{3} + \left(1 + \frac{(-1)^3}{3}\right)$$

$$= 2 - \frac{2}{3} = 4/3$$

Fundamental Theorem of Calculus

Two parts! I

II → justifies the above.



$$G(x) = \int_a^x f(s)\, ds$$

$$\sum_{j=1}^{10} j = \sum_{k=1}^{10} k$$

"area under the curve"

$$G(a) = 0$$

$$G(b) = \int_a^b f(s)\,ds$$

Claim: If $G(x) = \int_a^x f(s)\,ds$, then

$$G'(x) = f(x).$$

Suppose you can find some antiderivative $F(x)$ of $f(x)$.

$$F'(x) = f(x)$$

$$F(x) = G(x) + C$$

$$F(a) = \underbrace{G(a)} + C$$

$$= 0 + C$$

$$C = F(a)$$

$$F(x) = G(x) + F(a)$$

$$F(x) - F(a) = G(x)$$

$$F(b) - F(a) = G(b) = \int_a^b f(x)\,dx$$