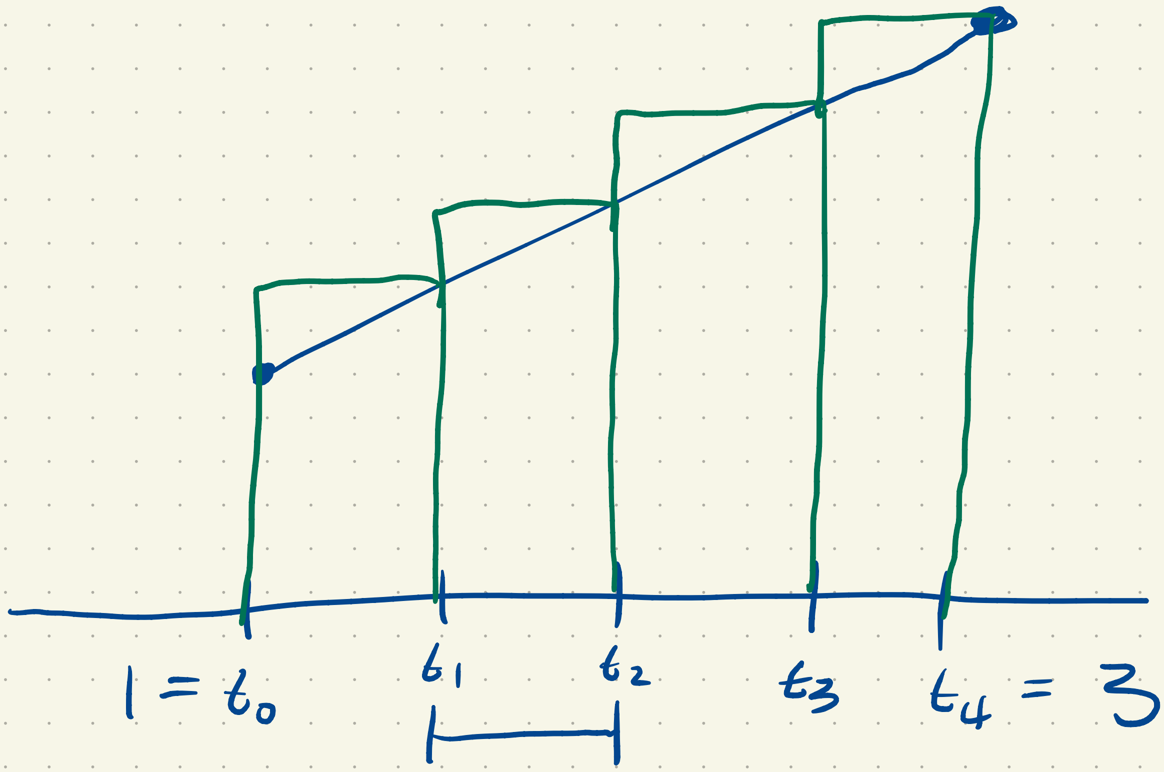
Last class

$v(t) = t$ m/s



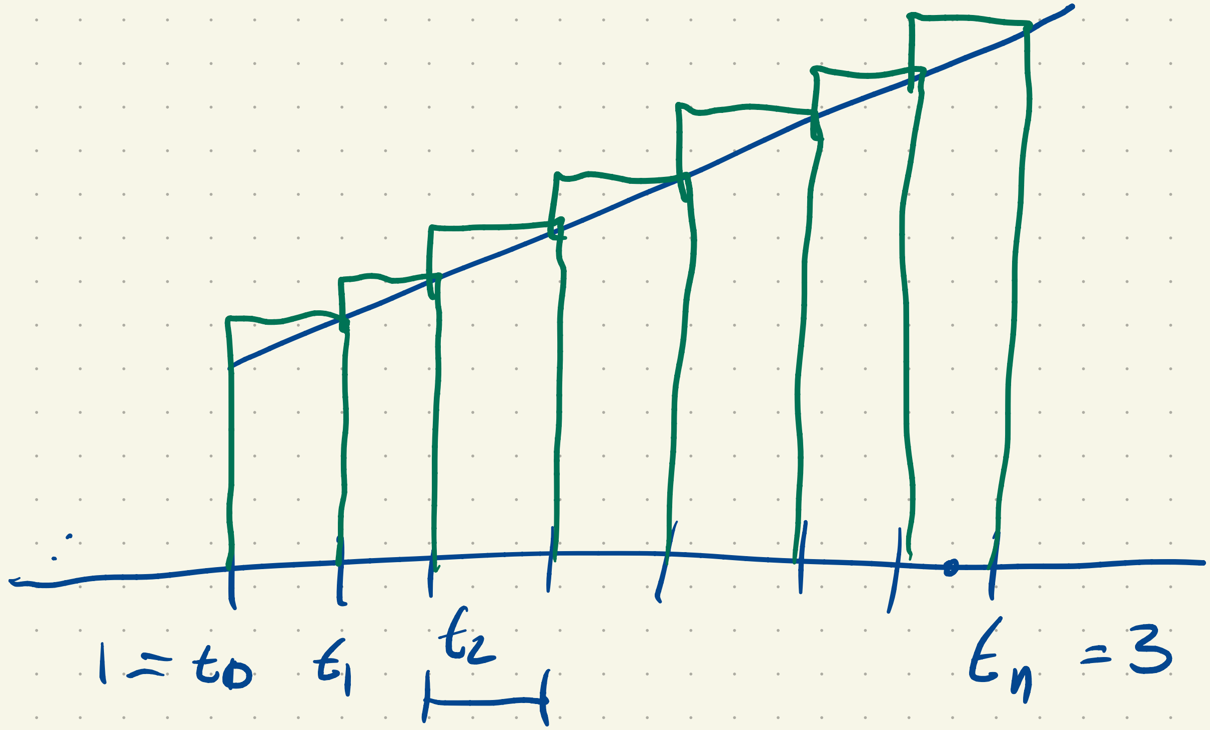
$\Delta t = \frac{1}{2}$

$$\sum_{k=1}^{4} v(t_k)\,\Delta t = 4.625$$

n subintervals

$4 + \frac{2}{n}$

$\int_a^b f(x)\,dx$



$\Delta t = \frac{3-1}{n} = \frac{2}{n}$

$t_k = 1 + k\,\Delta t$

$R_n = \sum_{k=1}^{n} \overbrace{v(t_k)\,\Delta t}$

est of dist. traveled over k'th subinterval

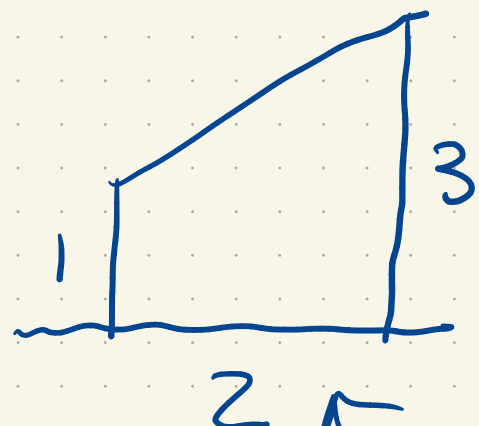
$v(t) = t \qquad v(t_k) = t_k$

Riemann sum

$$R_n = \sum_{k=1}^{n} v(t_k)\,\Delta t = \sum_{k=1}^{n} \left[1 + k\,\Delta t \right] \Delta t$$

$\Delta t = \frac{2}{n} \qquad \frac{2}{500}$

We expect $\lim_{n \to \infty} R_n = 4$



area is 4

(total distance run is 4 m)

$$\Delta t = \frac{2}{n}$$

$$\sum_{k=1}^{n} \left[1 + k \, \Delta t \right] \Delta t = \sum_{k=1}^{n} \Delta t + \sum_{k=1}^{n} k \, \Delta t^2$$

$$= \Delta t \sum_{k=1}^{n} 1 + \Delta t^2 \sum_{k=1}^{n} k$$

$$\sum_{k=1}^{n} 1 = \underbrace{1 + 1 + \cdots + 1}_{n}$$

$$= n$$

$$\sum_{k=1}^{n} k = 1 + 2 + 3 + \cdots + n$$

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$1+2+3+4+5 = 15$$

$$5+4+3+2+1 = 15$$

$$\underbrace{6+6+6+6+6}$$

$$5 \cdot 6 = 30$$

$$\sum_{k=1}^{n} k^2$$

$$\frac{100 \cdot 101}{2} \qquad 50 \cdot 101$$

$$5050$$

$$1+2+3+ \cdots + n = \frac{n(n+1)}{2}$$

$$n+(n-1)+(n-2)+ \cdots + 1 = \frac{n(n+1)}{2}$$

$$\underbrace{(n+1)+(n+1)+(n+1)+ \cdots + (n+1)}_{n} = n(n+1)$$

$$\Delta t \sum_{k=1}^{n} 1 + \Delta t^2 \sum_{k=1}^{n} k = \Delta t \cdot n + \Delta t^2 \frac{n(n+1)}{2}$$

$$= \Delta t \cdot n + \Delta t^2 \frac{(n^2+n)}{2}$$

$$\Delta t = \frac{2}{n}$$

$$n \Delta t = 2$$

$$= \Delta t \cdot n + (n\Delta t)^2 \frac{\left(1 + \frac{1}{n}\right)}{2}$$

$$= 2 + 4 \frac{\left(1 + \frac{1}{n}\right)}{2}$$

$$= 4 + \frac{2}{n}$$

$$R_n = 4 + \frac{2}{n}$$

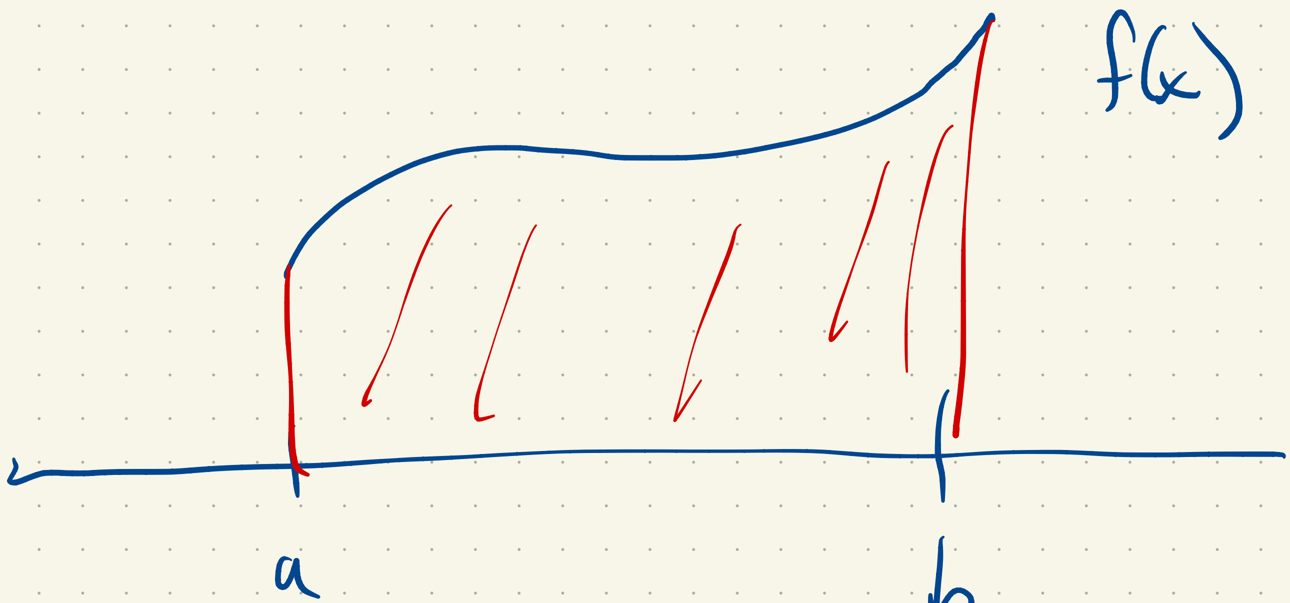
$$\lim_{n\to\infty} R_n = 4 + 0 = 4$$

area of region

net distance traveled,

$$\int_1^3 v(t)\,dt$$

Definite Integral:

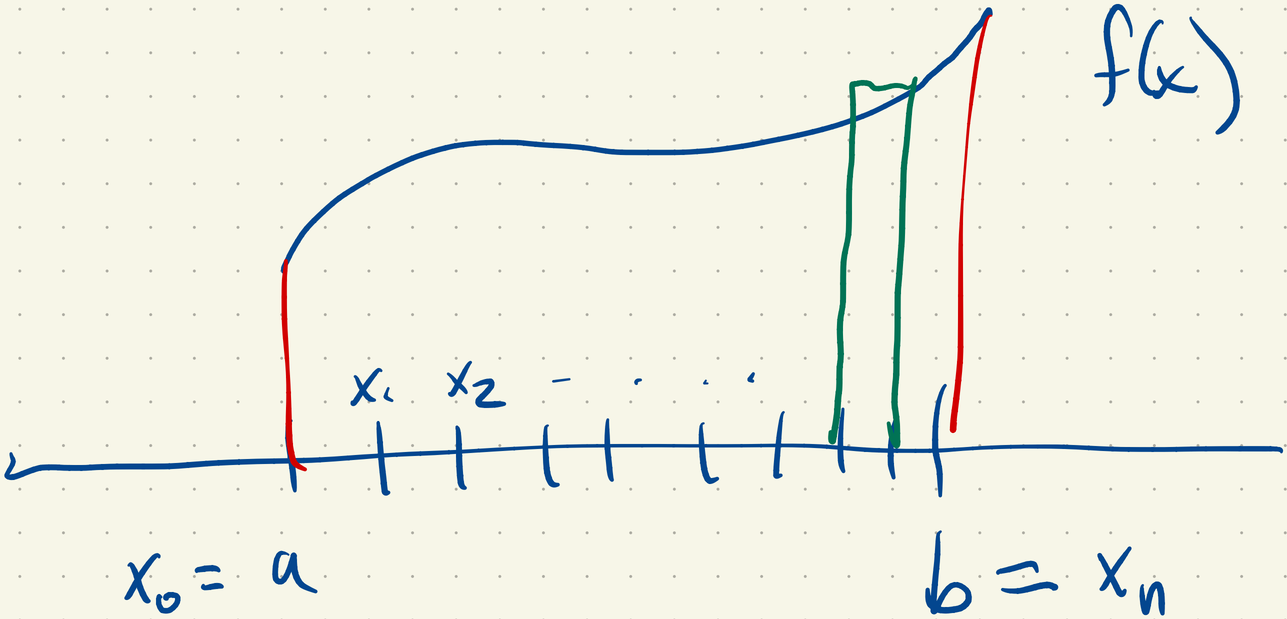


$$\int_a^b f(x)\,dx \leftarrow \text{number}$$

a) area (signed)

b) change (net)

x_k^*

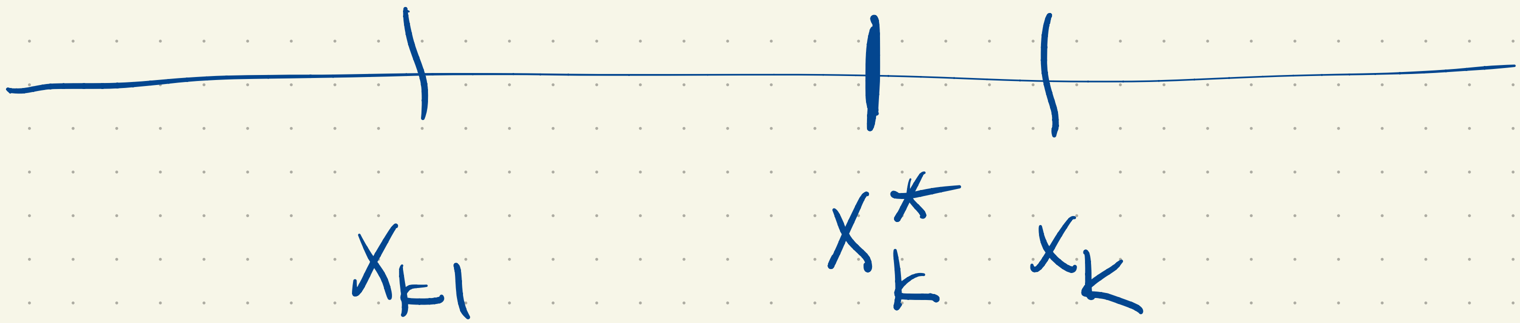


$\left(\frac{3-1}{n}\right)$

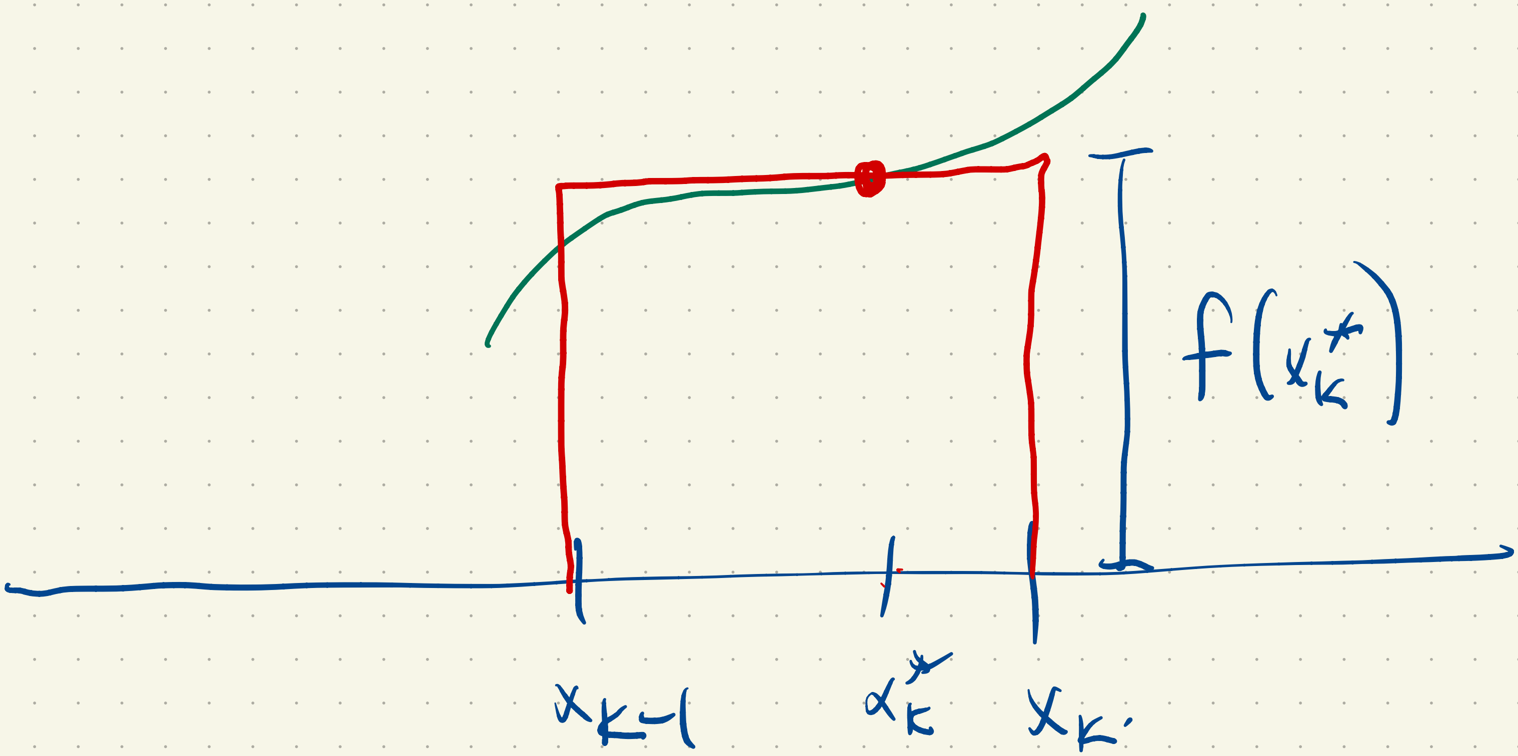
n subintervals of length $\Delta x = \frac{b-a}{n}$

$x_k = x_0 + k\Delta x$ $\quad$ $x_n = a + n \cdot \left(\frac{b-a}{n}\right) = b$

Pick sample points x_k^* in the k^{th} subinterval



x_{k-1}
x_k^*
x_k



$f(x_k^*)$
x_{k-1}
x_k^*
x_k

$$R_n = \sum_{k=1}^{n} f(x_k^*)\Delta x$$

n'th Riemann sum

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} R_n$$

so long as a) the limit exists

b) the limit does not depend on choice of sample points.

$$\underbrace{\int_1^3 v(t)\,dt}$$

→ net distance traveled by vole

$$\underbrace{\int_a^b f(x)\,dx}$$

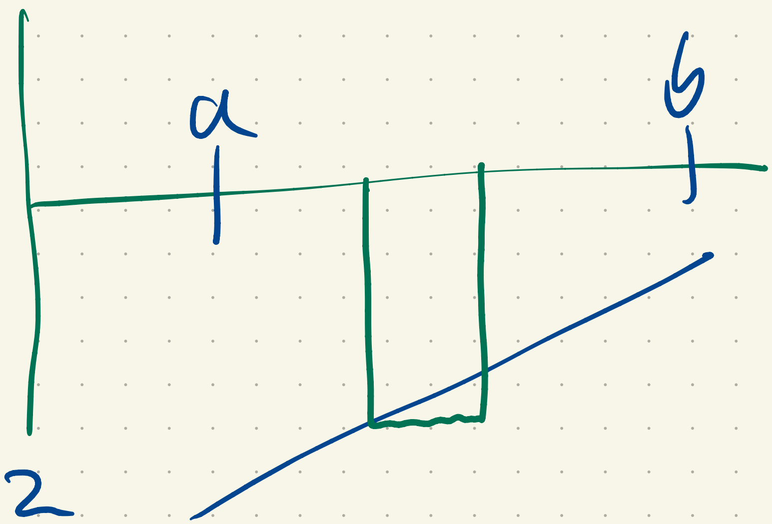
definite integral

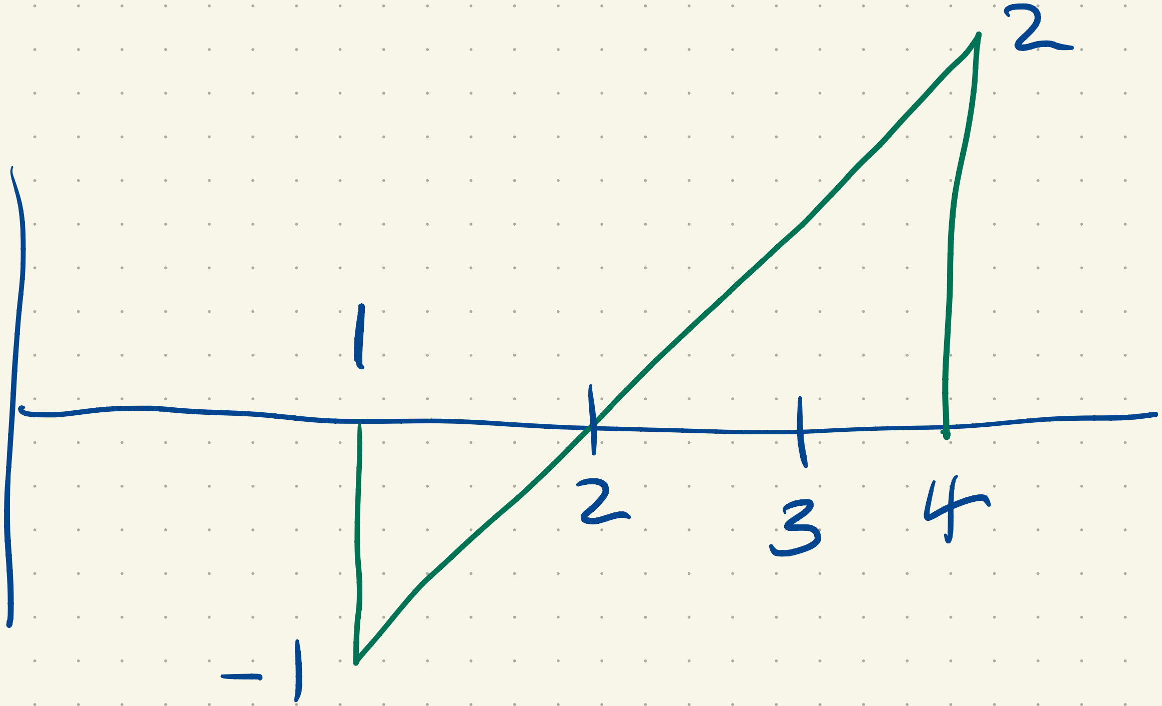
exists if a) $f(x)$ is cts.

b) $f(x)$ is bounded and discontinuous at only finitely many points.

$$R_n = \sum_{k=1}^{n} f(x_k^*)\Delta x$$

$f(x) < 0$ everywhere





$v(t) = -2 + t$

$1 \le t \le 4$

$$\int_1^4 v(t)\,dt = -\frac{1}{2} + 2 = 3/2$$

Ant velocity is $v(t) = -2 + t$ cm/s

$1 \le t \le 4$

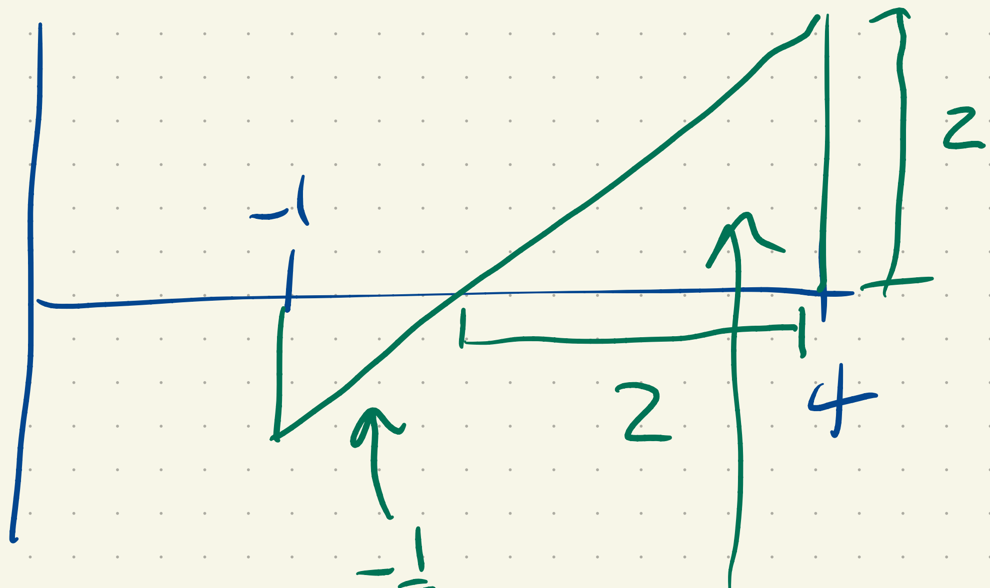
$v(1) = -1$

$v(2) = 0$

$v(3) = 1$

$v(4) = 2$ cm/s

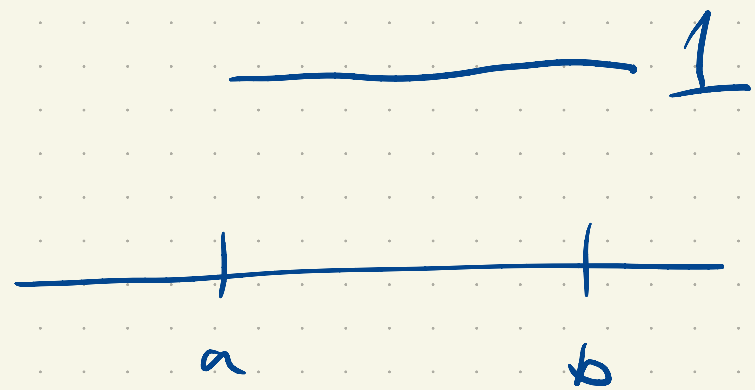
$\uparrow$ s



$\frac{1}{2} 2 \cdot 2 = 2$

$$\int_1^4 v(t)\,dt = -\frac{1}{2} + 2 = \frac{3}{2}$$

Properties



a) $$\int_a^b 1\,dx = b - a$$

b) $$\int_a^b c\,f(x)\,dx = c\int_a^b f(x)\,dx$$

c) $$\int_a^b \big(f(x) + g(x)\big)\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

linearity